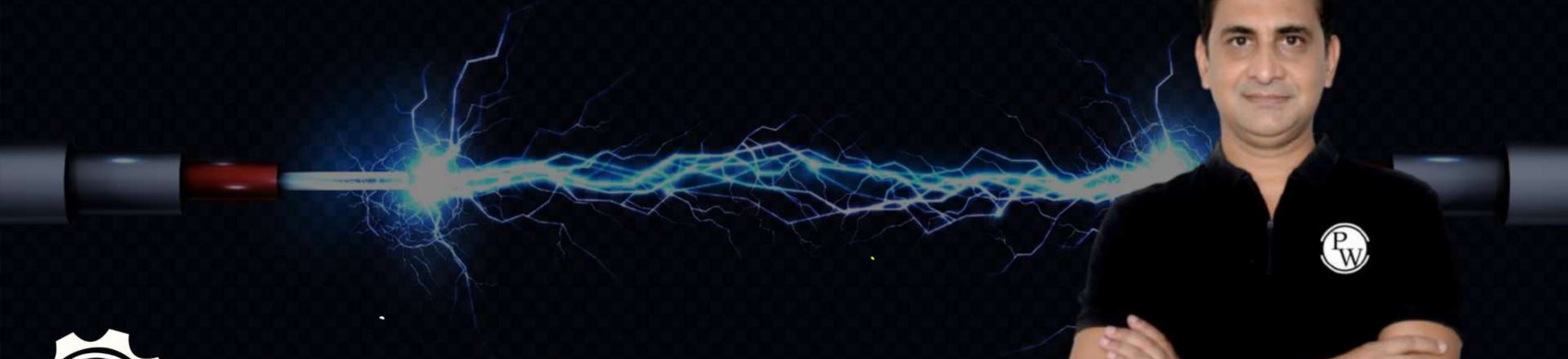


Electrical Engineering



Power System



Lecture No 1 Economic Load Dispatch



By- Ajay Gupta Sir



Recap of Previous class

1. Fast decoupled Method



Topics to be Covered

1. Different cases of ELD

2. ELD with Losses.

3. Loss Coefficient.

ECONOMIC LOAD DISPATCH

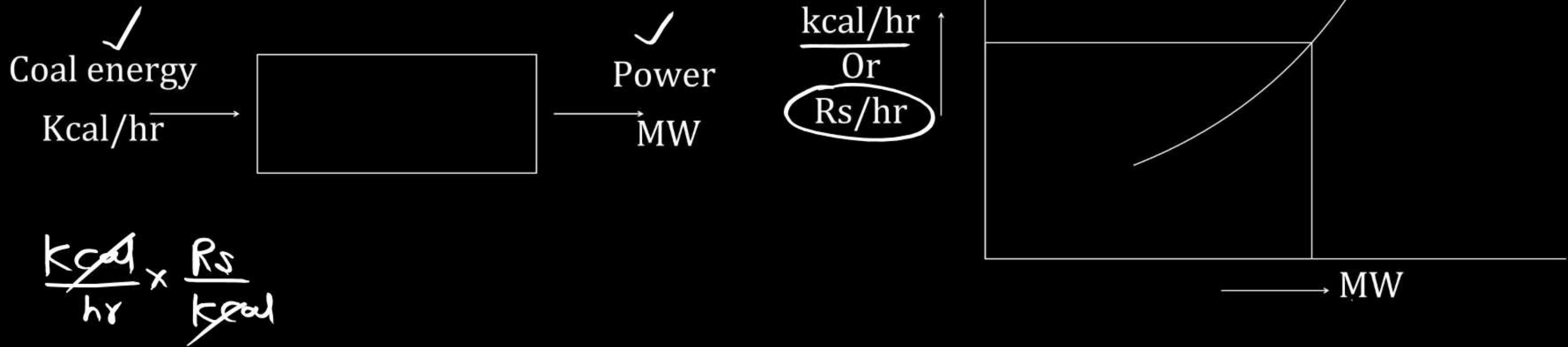
Economic Load Dispatch :

1 Jelen
+ 151.

It involve solution of two problem.

- Selection of source optimally out of available source to meet the expected demand with sufficient reserve. It is called unit commitment.
- Distribute the load among the generating unit in such a manner so that cost of power will be minimum.
- In economic load dispatch we do not consider the fixed cost is capital cost.
- We will consider only variable cost. The main variable cost is fuel cost & cost of labor, maintenance & water cost.
- We take labor, supplies maintenance cost and water cost as a ~~some~~ percentage of fuel cost.

Input – Output Curve:



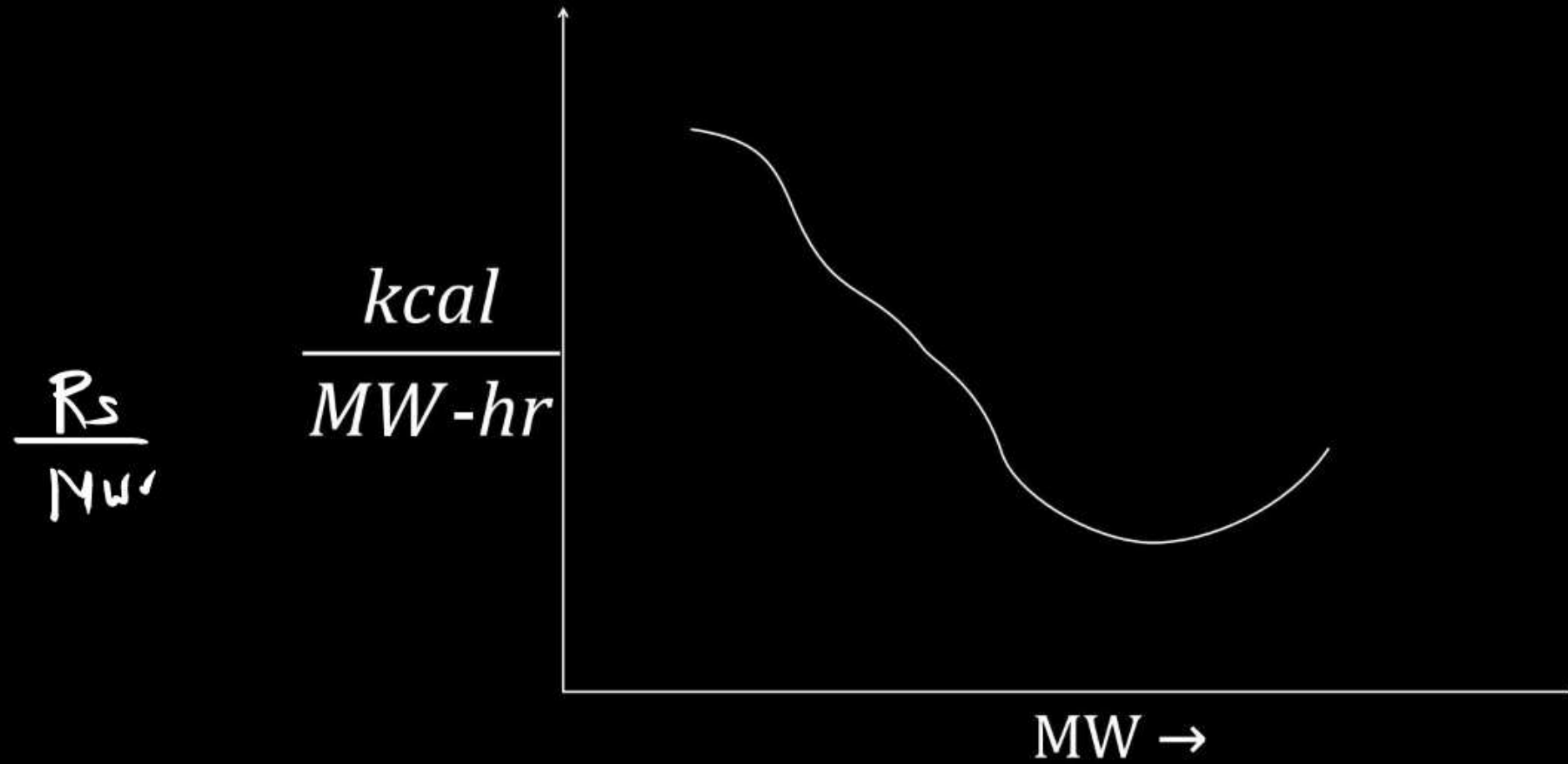
- It is a curve between fuel input in kcal/hr v/s output power.
- Fuel input can be converted into Rs/hr by multiplying kcal/hr by cost of fuel in Rs/kcal.

$$\frac{\text{kcal}}{\text{hr}} \times \frac{\text{Rs}}{\text{kcal}} = \text{Rs / hr}$$

Heat rate Curve:



- It is a curve between heat rate kcal/MW-hr v/s output power in MW



Incremental Fuel Cost:

- Ratio of small change of input to small change of output.

$$I.C = \frac{\Delta I / P}{\Delta O / P} = \frac{d(\text{Input})}{d(\text{output})} = \frac{dF}{dP}$$

$$\frac{\text{kcal}}{\text{MWhr}} \text{ or } \boxed{\frac{\text{Rs}}{\text{MW} - \text{hr}}}$$

$$\text{Incremental efficiency} = \frac{dP}{df}$$



Method of Loading Turbo Generator:



1. Base load to capacity

1	2 ↓	3 ↓	4 ↓
88% (F.L.)	96% (F.L.)	92% (F.L.)	95% (F.L.)
<u>150 MW</u>	<u>200 MW</u>	<u>300 MW</u>	<u>100 MW</u>

✓✓ 400 MW $\Rightarrow$ Demand.

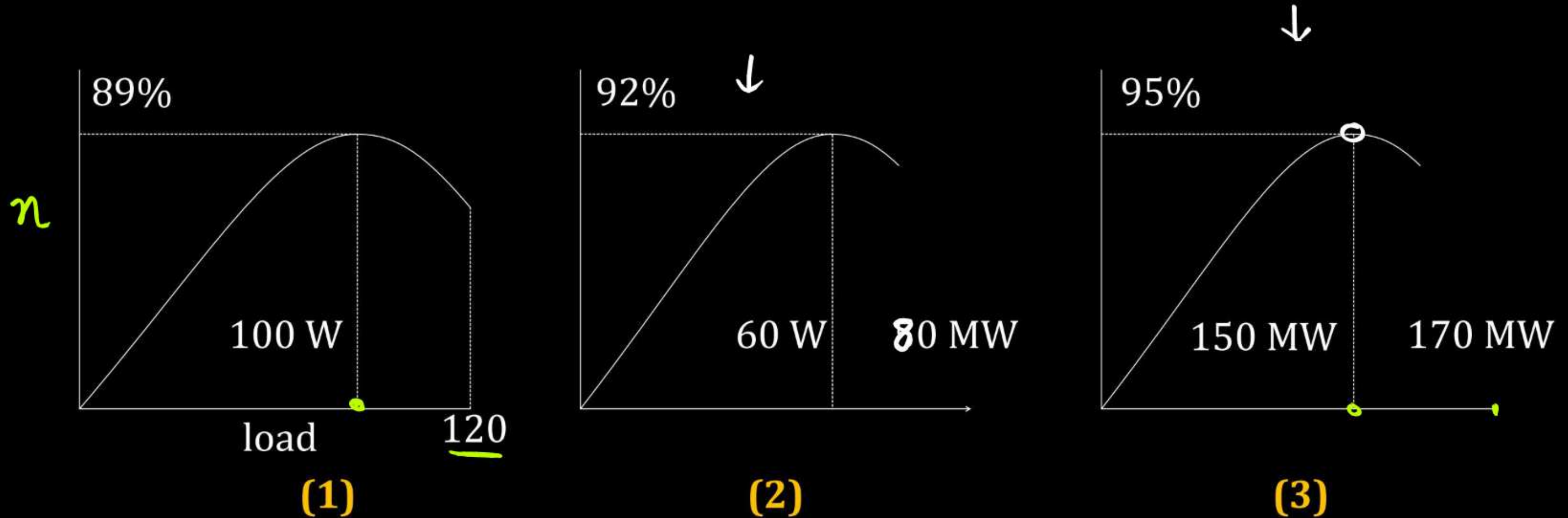
Unit 2 – 200 MW ✓

Unit 4 – 100 MW ✓

Unit 3 – 100 MW ✓

- The turbo unit are loaded to full capacity one after other in order of their efficiency.

2. Base load to most efficient load.





Demand 300 MW ✓

Unit 3 → 150 MW ✓

Unit 2 → 60 MW

Unit 1 → 90 MW ✓

The turbo unit are loaded up to point of maximum efficiency in order of their efficiency.

3. Loading proportional to their rating.

	1	2	3	
Rating	100 MW	200 MW	300 MW	$= 600 \text{ MW}$
Demand	300 MW	✓		
Unit 1	50 MW	✓		
Unit 2	100 MW	✓		
Unit 3	150 MW	✓		

4 Equal incremental Cost operation :-

The objective of economic load scheduling is to minimize total cost of generation subjected to following constraint

$$P_1 + P_2 + P_3 = P_D$$

Equality Constraint

$$\sum_{n=1}^n P_n = P_D \text{ (If losses is neglected)}$$

Inequality Constraint

$$P_{\min} \leq P_i \leq P_{\max}$$

Economical Dispatch Neglecting Losses:

(a) Two generating unit:

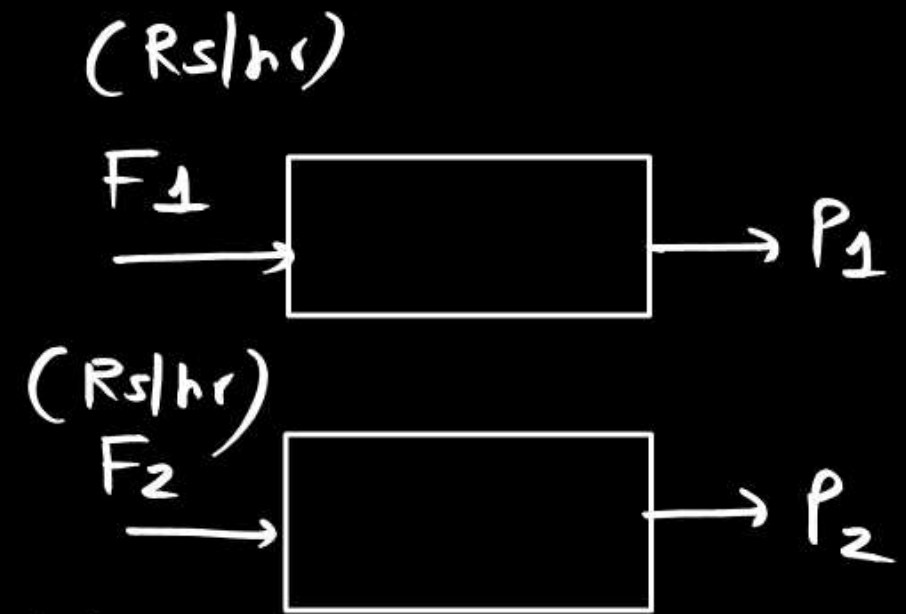
Suppose generating cost of two unit is

Unit 1 → For output power of P_1 cost in F_1

Unit 2 → For output power of P_2 cost is F_2

Total power $\underline{P_D} = \underline{P_T} = \underline{P_1} + \underline{P_2}$ (equality constraint)

Total cost $\underline{F_T} = \underline{F_1} + \underline{F_2}$ (Min)



$$P_T = P_1 + P_2 = \text{Constant}$$

$$dP_1 + dP_2 = 0$$

$$dP_1 = -dP_2 \quad \text{--- ①}$$

$$P_D = 400 \text{ MW (constant)}$$

$$F_T = F_1 + F_2$$

$$\frac{dF_T}{dP_1} = 0$$

$$\frac{dF_1}{dP_1} + \frac{dF_2}{dP_1} = 0$$

From eqⁿ (1)

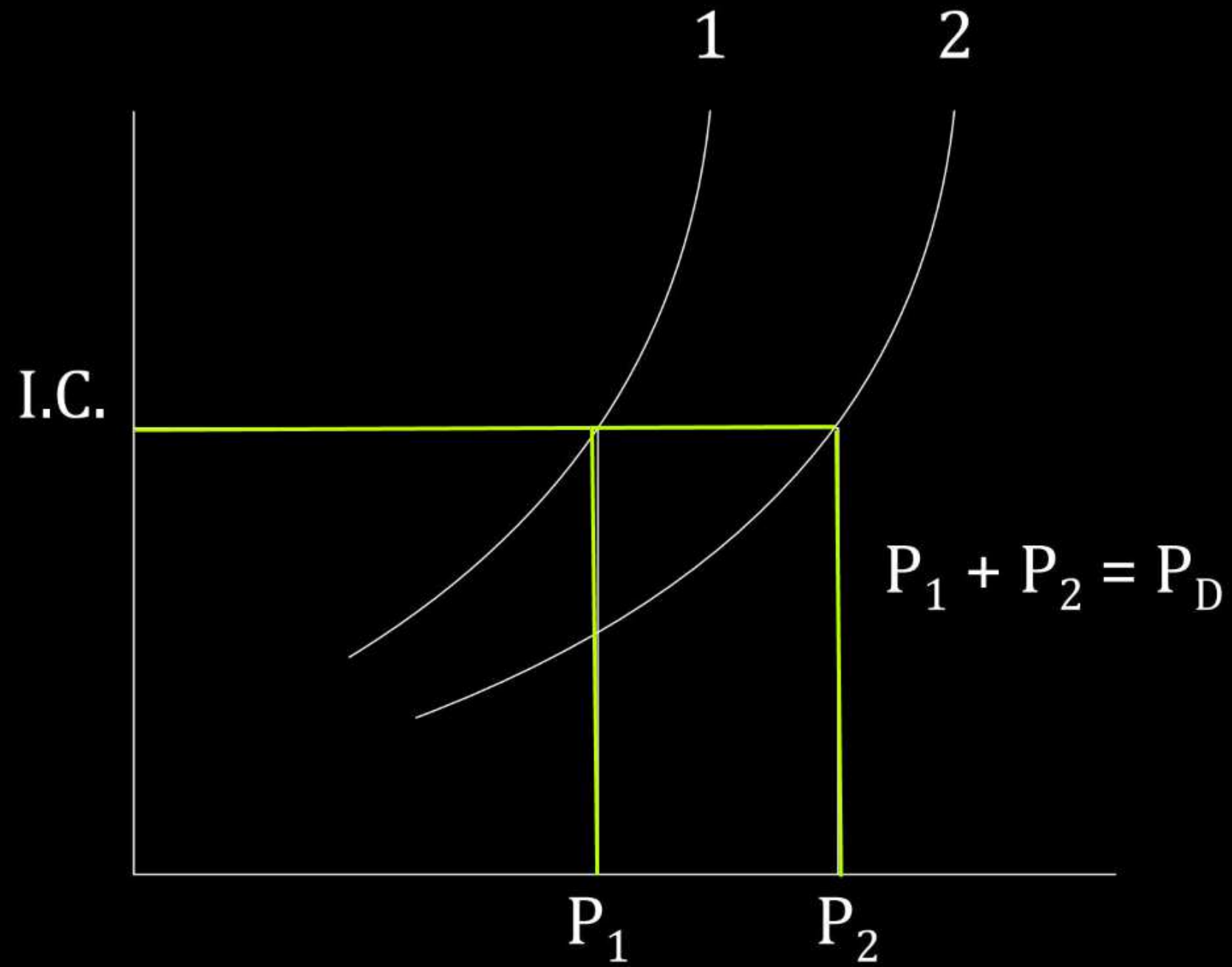
$$\frac{dF_1}{dP_1} + \frac{dF_2}{(-dP_2)} = 0$$

$$\boxed{\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}}$$

$$\boxed{(IC)_1 = (IC)_2}$$

For economic load scheduling, the incremental cost of both unit should be same

||

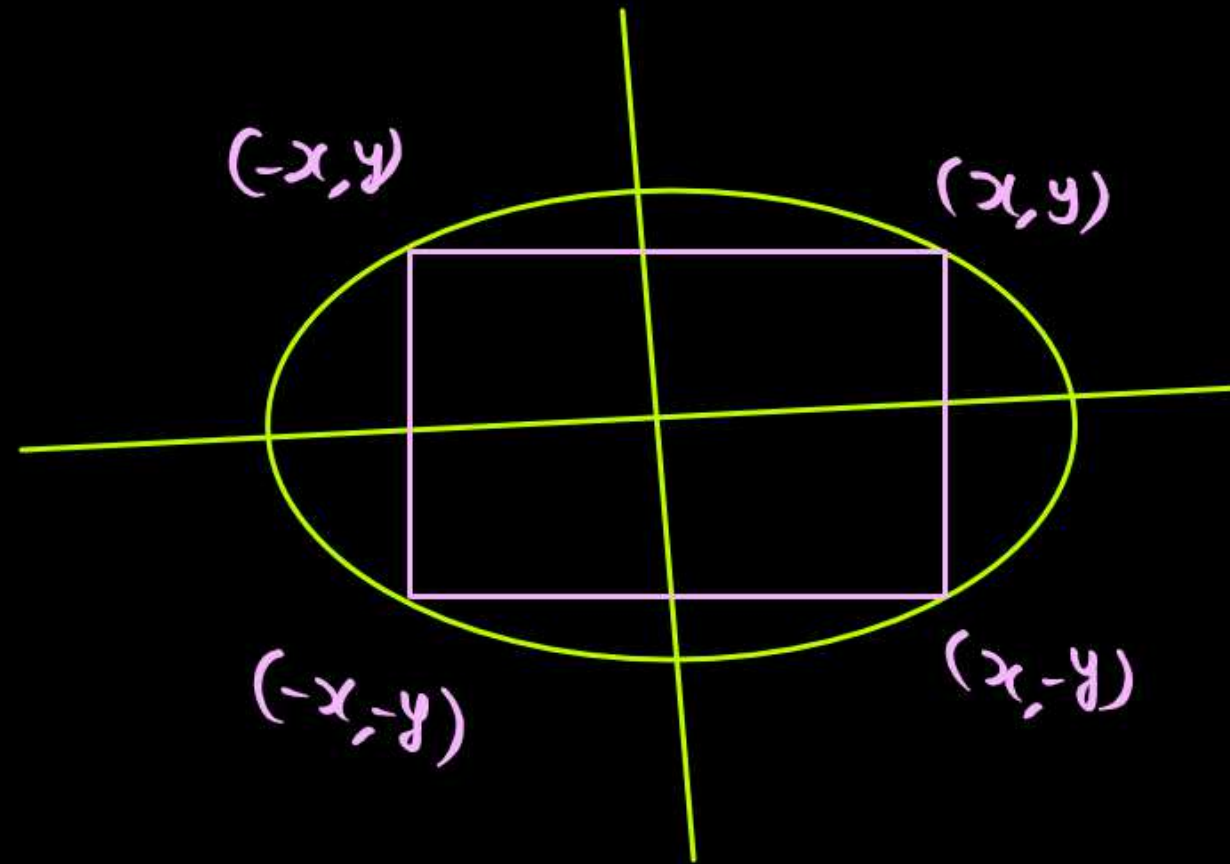


Method of Lagrange's Multiplier:



This method is used to solve maxima and minima of a function satisfying a equality constraint.

Question: Find maximum area of a rectangle that can be inscribe inside the ellipse



$$\begin{aligned} \text{Area} &= (2x)(2y) \\ &= 4xy \end{aligned}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Area} = 4xy$$

(Main function)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(equality constraint)

Auxiliary function = Main function + λ (equality constraint)

$$F = 4xy + \lambda \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)$$

$$\frac{dF}{dx} = 0$$

$$x = \frac{a}{\sqrt{2}}$$

$$\frac{dF}{dy} = 0$$

$$y = \frac{b}{\sqrt{2}}$$

$$\begin{aligned} \text{Area} = 4xy &= 4 \left(\frac{a}{\sqrt{2}} \right) \left(\frac{b}{\sqrt{2}} \right) \\ &= \underline{2ab} \end{aligned}$$

(b) If large number of generating unit exist:

$$P_D = P_T = P_1 + P_2 + P_3 \dots + P_n \text{ (equality constraint)} \quad (1)$$

$$F_T = F_1 + F_2 + F_3 \dots + F_n$$



➤ The above problem is having an equality constraint as given by equation (1). So, it can be solved by using simplex method of optimization technique. Which say construct an auxiliary function (Lagrangian function) in term of Main function and constraint.

Auxiliary function = Main function + λ (Constraint)



$$F = F_T + \lambda (P_D - \sum P_n)$$



The main function will be minimum, when auxiliary function is minimum.



$$F = F_T + \lambda \left(P_D - \sum_{n=1}^n P_n \right) \quad P_D = \text{constant}$$

$$F = F_1 + F_2 + \dots + F_n + \lambda \left[P_D - (P_1 + P_2 + \dots + P_n) \right]$$

$$\frac{dF}{dP_1} = \frac{dF_1}{dP_1} + \frac{dF_2}{dP_1} \dots + \frac{dF_n}{dP_1} + \lambda (0 - 1) = 0$$

$$\frac{dF_1}{dP_1} + 0 - \lambda = 0 \quad \Rightarrow \quad \lambda = \frac{dF_1}{dP_1} \quad - (1)$$

Similarly

$$\frac{dF}{dP_2} = 0 \Rightarrow \frac{dF_1}{dP_2} + \frac{dF_2}{dP_2} + \dots + \frac{dF_n}{dP_2} + \lambda(0-1) = 0$$

$$\frac{dF_2}{dP_2} - \lambda = 0$$

$$\lambda = \frac{dF_2}{dP_2} \quad \text{--- (2)}$$

Similarly

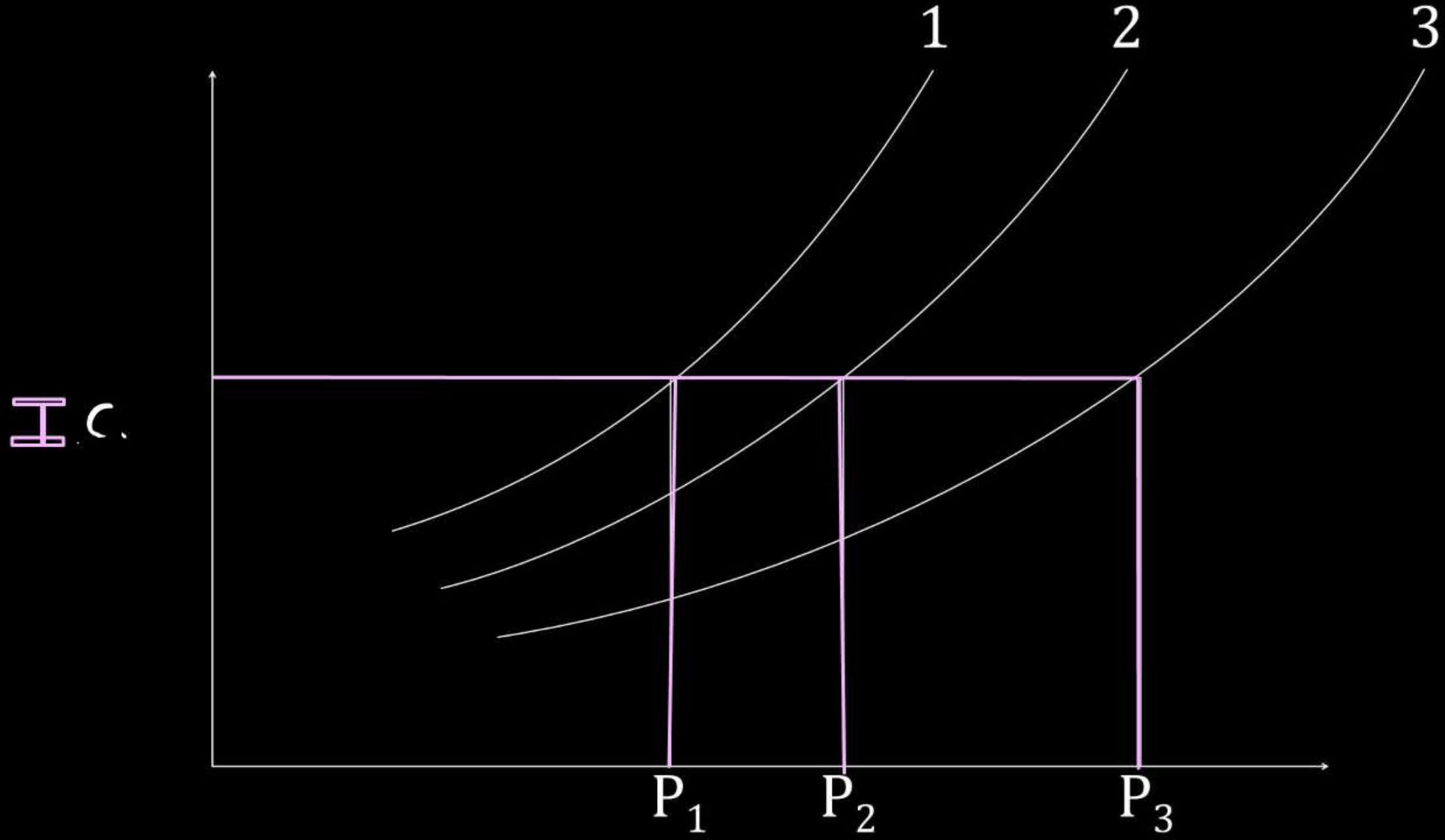
$$\frac{dF}{dP_3} = 0$$

$$\lambda = \frac{dF_3}{dP_3} \quad \text{--- (3)}$$

From eqn ①, ② ③

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_3}{dP_3} = \dots = \lambda$$

For economic load scheduling, the incremental cost of all the unit should be same





My son was saying
that Economic Load
Dispatch is a very
tough topic.

No, it's actually very easy.
Tell him, just like you purchase
items from the market using
common sense, the same
common sense you use in
market, you will use in
Economic Load Dispatch
and you can do it.



The incremental cost characteristic of two generator delivering 200 MW are as follows

$$\frac{dF_1}{dP_1} = \underline{20} + \underline{0.1} P_1 \quad \dots(1)$$

$$\left. \frac{Rs}{Mw \text{ hr}} \right]$$

$$\frac{dF_2}{dP_2} = 16 + 0.2 P_2 \quad \dots(2)$$

For economic operation the generation P_1 & P_2 should be ...

$$P_1 + P_2 = 200 \quad \dots(1)$$

$$(IC)_1 = (IC)_2$$

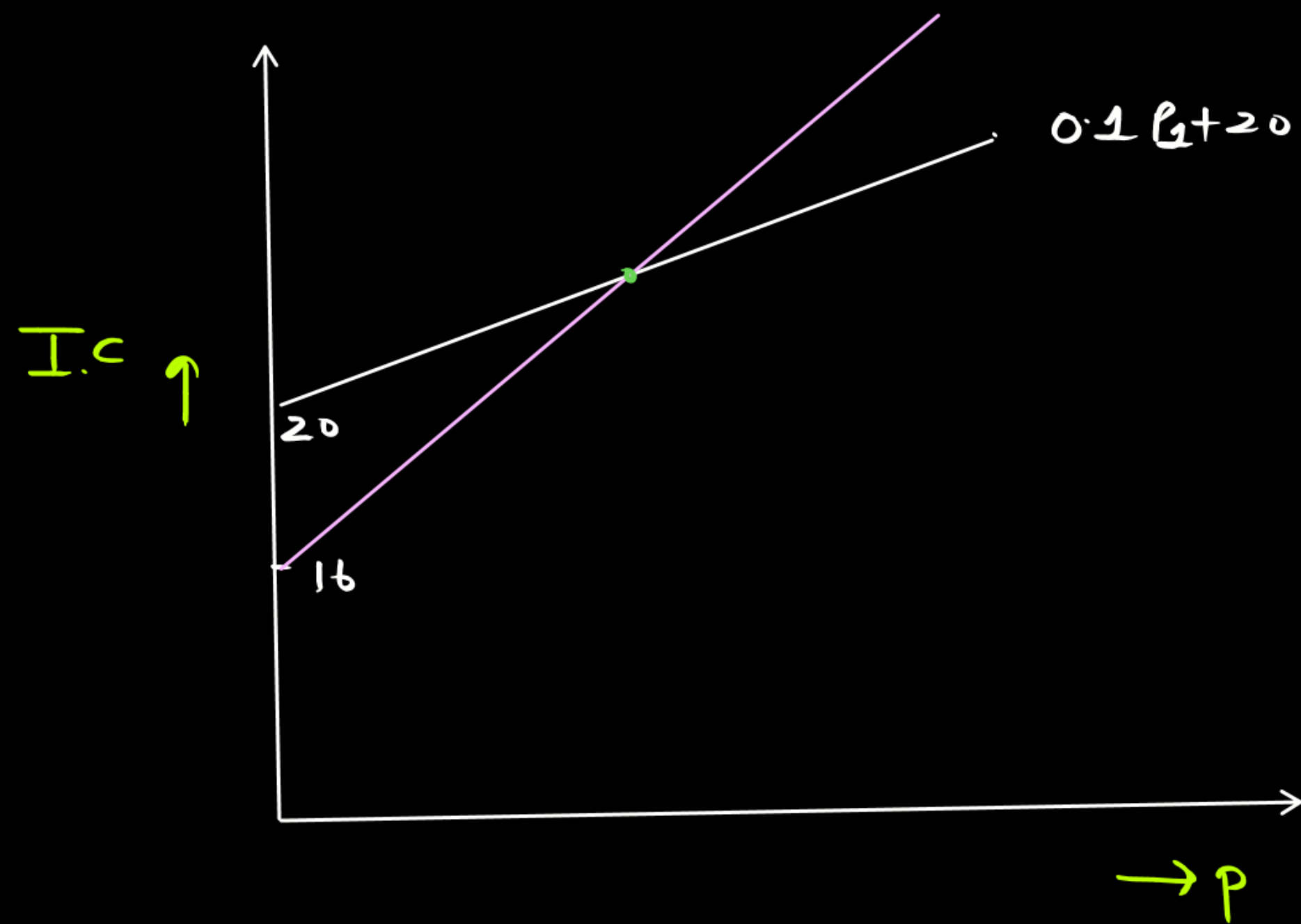
$$20 + 0.1 P_1 = 16 + 0.2 P_2$$

$$0.1P_1 - 0.2P_2 = -4$$

$$P_1 - 2P_2 = -40 \quad (2)$$

$$\begin{array}{r} P_1 + P_2 = 200 \\ P_1 - 2P_2 = -40 \\ \hline + \quad + \\ \hline 3P_2 = 240 \\ P_2 = 80 \end{array}$$

$$\begin{array}{l} P_1 + P_2 = 200 \\ 80 + P_2 = 200 \\ P_2 = 120 \end{array}$$



Case-I :

- If power calculate by economic operation violate the active power limit of unit. Then we will fix the power to that limit & remaining power will be given by another unit.

$$P_1(cal) \geq P_1(Max)$$

$$P_1 = \underline{P_1(Max)}$$

$$\left(\begin{array}{l} P_1 = 80 \\ \underline{90} < P_1 < \underline{180} \end{array} \right.$$

$$50 < P_1 < 70$$

Q.



A power system has two generator with following cost curve.

$$F_1 = 0.006 P_1^2 + 8 P_1 + 360$$

(cost)

$$F_2 = 0.006 P_2^2 + 7 P_2 + 400$$

(cost)

The generator limit,

$$100 < P_1 < 650$$

$$50 < P_2 < 500$$

If load demand is 600 MW. Determine optimal generator of each generator.

$$(Ic)_1 = \frac{dF_1}{dP_1} = 0.012 P_1 + 8 \quad - (1)$$

$$(Ic)_2 = \frac{dF_2}{dP_2} = 0.012 P_2 + 7 \quad - (2)$$

$$(Ic)_1 = (Ic)_2$$

$$0.012 P_1 + 8 = 0.012 P_2 + 7$$

$$0.012 P_1 - 0.012 P_2 = -1$$

$$P_1 - P_2 = -83.33 \quad - \textcircled{3}$$

$$P_1 + P_2 = 600 \quad - \textcircled{4}$$

$$2P_1 = 516.67$$

$$P_1 = 258.335$$

$$P_2 = 600 - P_1$$

$$= 600 - 258.335$$

$$= 341.665$$

Q. In a power system fuel input per hour of plant 1 & 2 is as below.

$$F_1 = 0.20 P_1^2 + 30 P_1 + 100 \text{ Rs/hr}$$

$$F_2 = 0.25 P_2^2 + 40 P_2 + 80 \text{ Rs/hr}$$

The limit of generator,

$$\begin{cases} 20 \leq P_1 \leq 80 \\ 40 \leq P_2 \leq 200 \end{cases}$$

$P_1 =$

If total demand is 130 MW. Find economic operating schedule

$$(IC)_1 = \frac{dF_1}{dP_1} = 0.4 P_1 + 30$$

$$(IC)_2 = \frac{dF_2}{dP_2} = 0.5 P_2 + 40$$

$$(IC)_1 = (IC)_2$$

$$0.4P_1 + 30 = 0.5P_2 + 40$$

$$0.4P_1 - 0.5P_2 = 10$$

$$4P_1 - 5P_2 = 100 \quad \text{--- ①}$$

$$5P_1 + 5P_2 = 650$$

$$9P_1 = 750$$

$$P_1 = 83.33 \text{ MW}$$

$$P_2 = 130 - 80 = \underline{50}$$

$$P_1 + P_2 = 130 \quad \text{--- ②}$$

$$20 \leq P_1 \leq 80 \text{ MW}$$

$$P_1(\text{cal}) \geq P_2(\text{Max})$$

$$\boxed{P_1 = 80}$$

Case-II :



- If (Max I.C.) of unit 1 is less than (Minimum I.C.) of unit number 2. Then we load unit number 1 to its rated capacity first & remaining power we will take from unit number 2.

POTATO RATE

FROM ₹10/KG

TO ₹20/KG

120/8

TOTAL POTATOES
AVAILABLE

50 KG

POTATO RATE

FROM ₹25/KG ✓

TO ₹30/KG

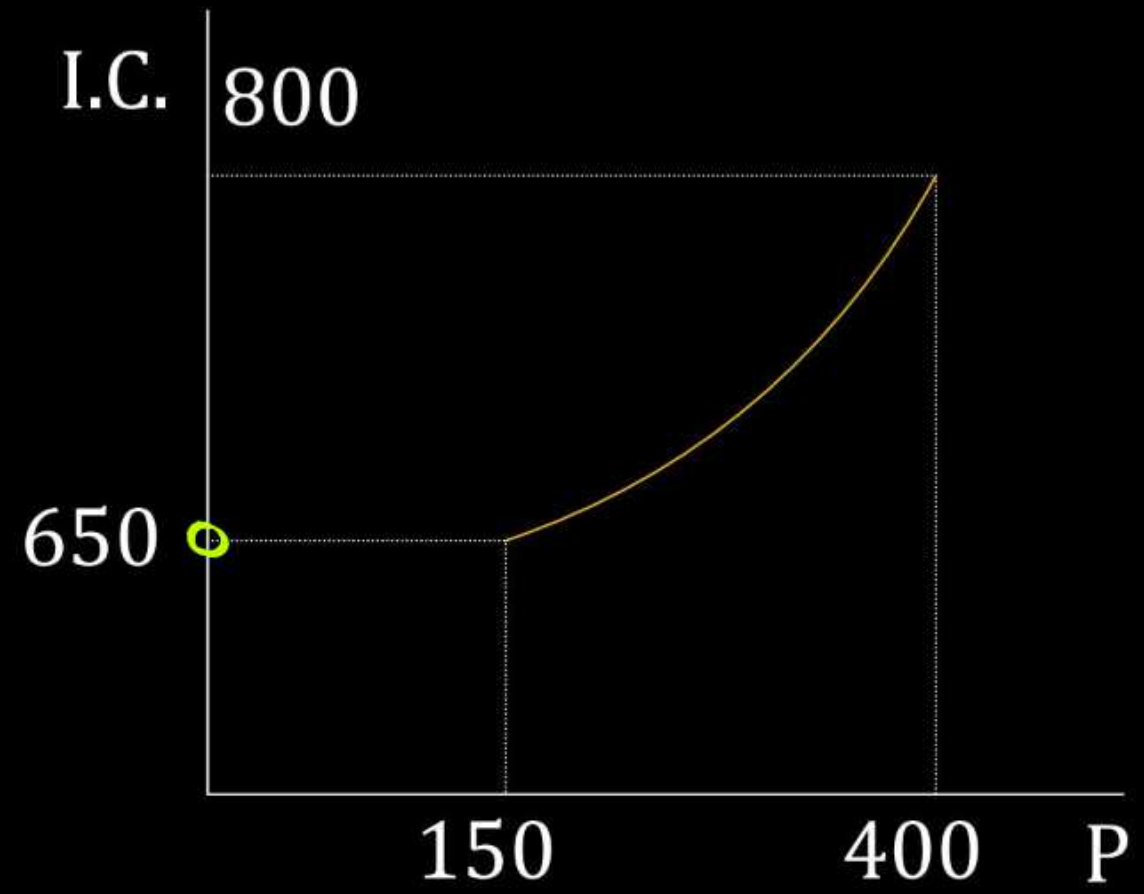
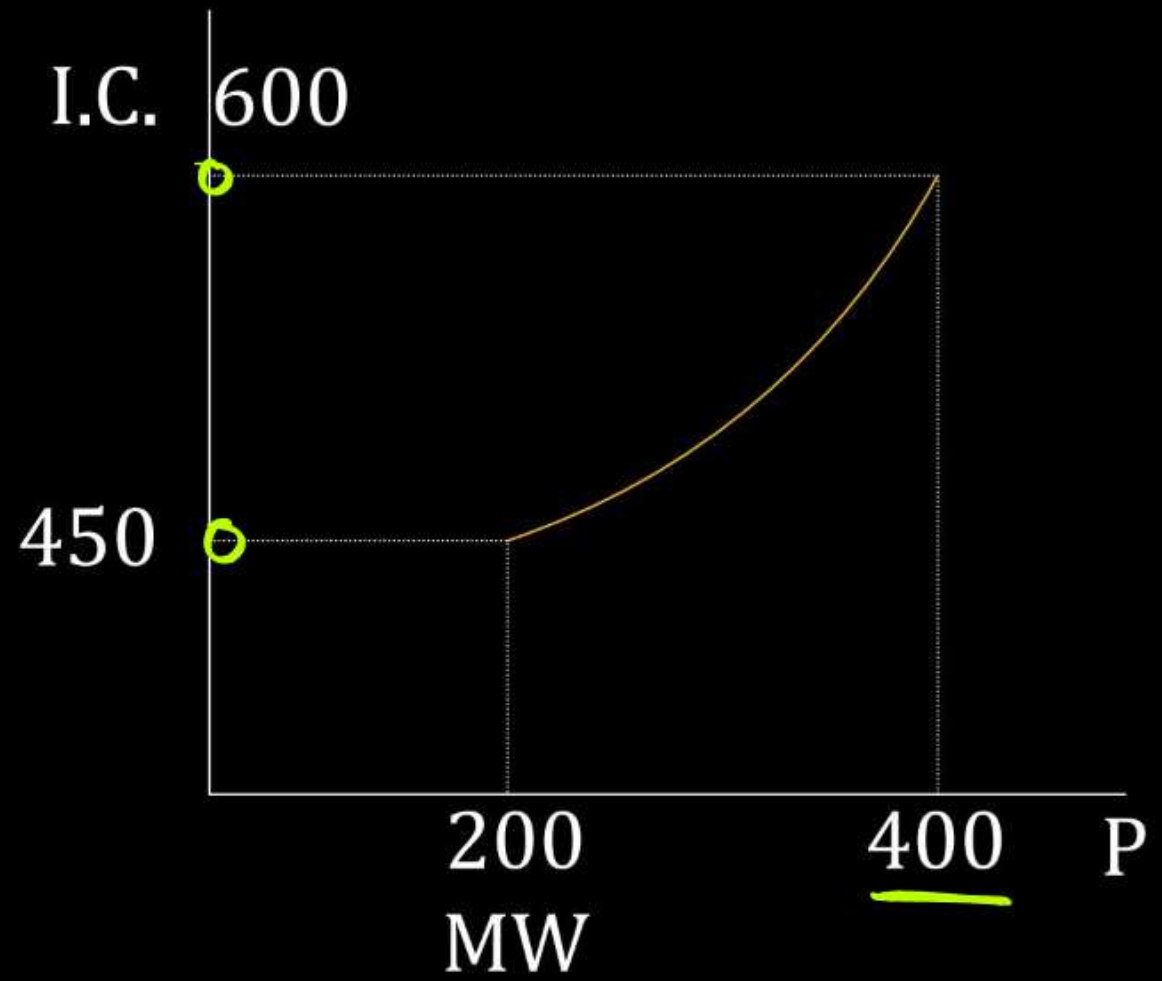
TOTAL POTATOES
AVAILABLE

100 KG

Q.



The incremental cost curve in Rs/hr for 2 generator supplying a common load of 700 MW as shown in figure.



The optimal schedule is $P_1 + P_2 = 700$ MW

Since Maximum I.C. of unit 1 < Minimum I.C. of unit 2

We load unit 1 up to its rated capacity.

$$P_1 = 450 \text{ (up to its rated Capacity)}$$

$$P_2 = 750 - 450 = 300$$

Case-III :



- If I.C. of any unit is fixed and it does not depend on power. Suppose if this cost is less than Minimum I.C. of other units. Then we will load the unit up to its rated capacity and remaining power will be taken by other units.

POTATO SHOP - 1

RATE

₹10 /KG

TOTAL AVAILABLE POTATOES
50 KG

120

50

FRESH
POTATOES

POTATO SHOP - 2

RATE

₹15 /KG to ₹20 /KG

TOTAL AVAILABLE POTATOES
70 KG

POTATO SHOP - 3

RATE

₹17 /KG to ₹30 /KG

TOTAL AVAILABLE POTATOES
OVER 100 KG

70kg

Q.



Incremental cost of 3 generating units are as given

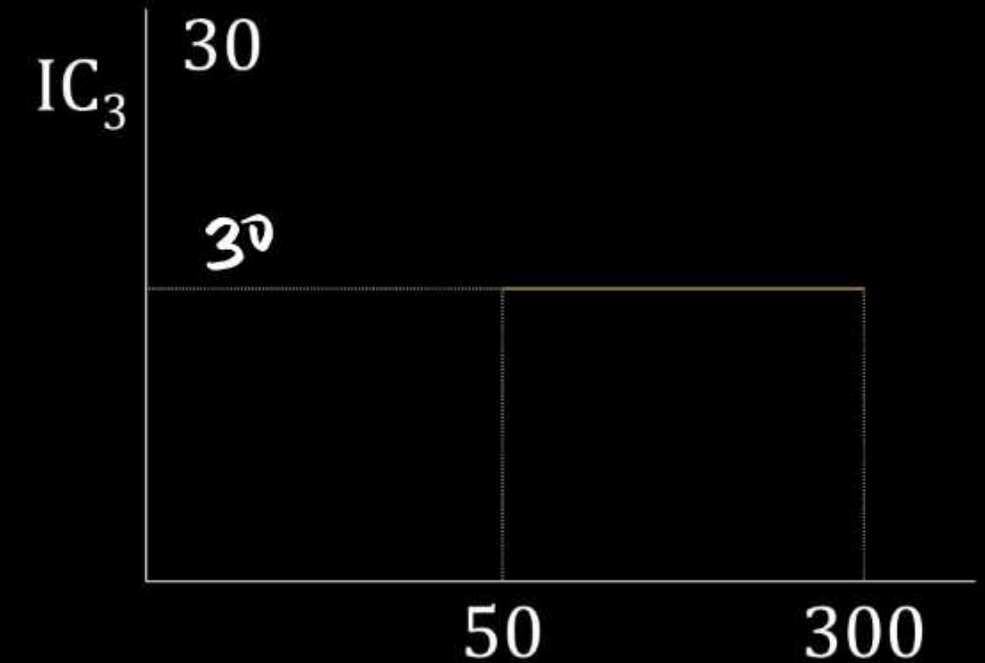
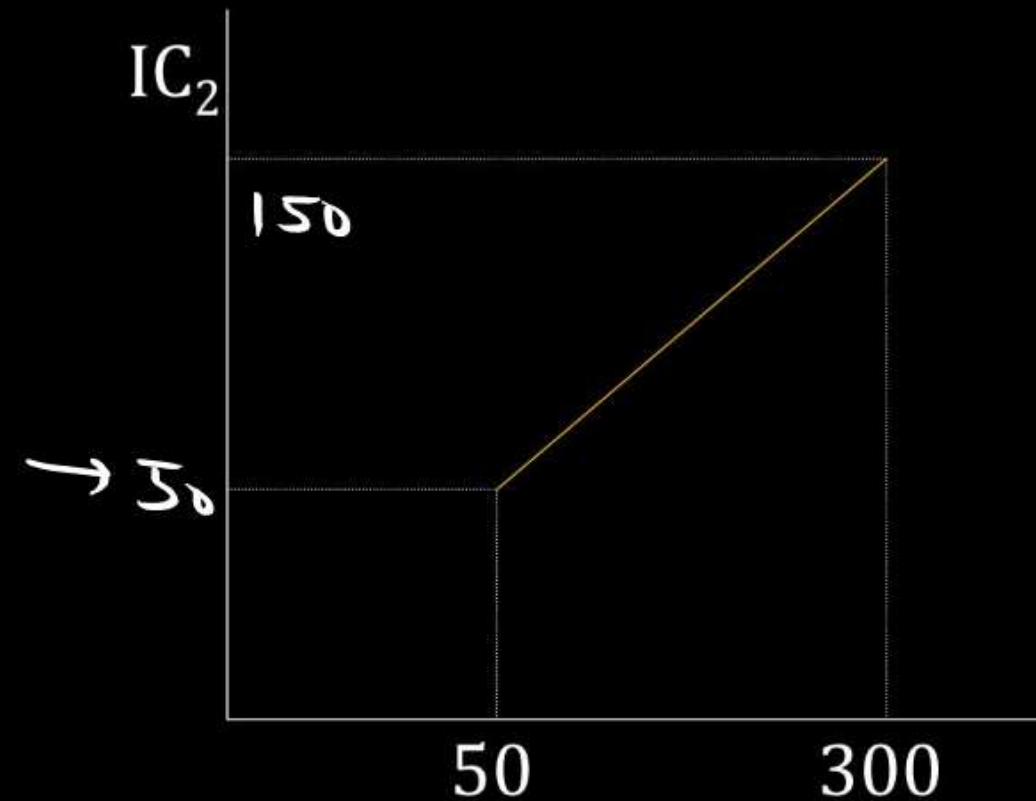
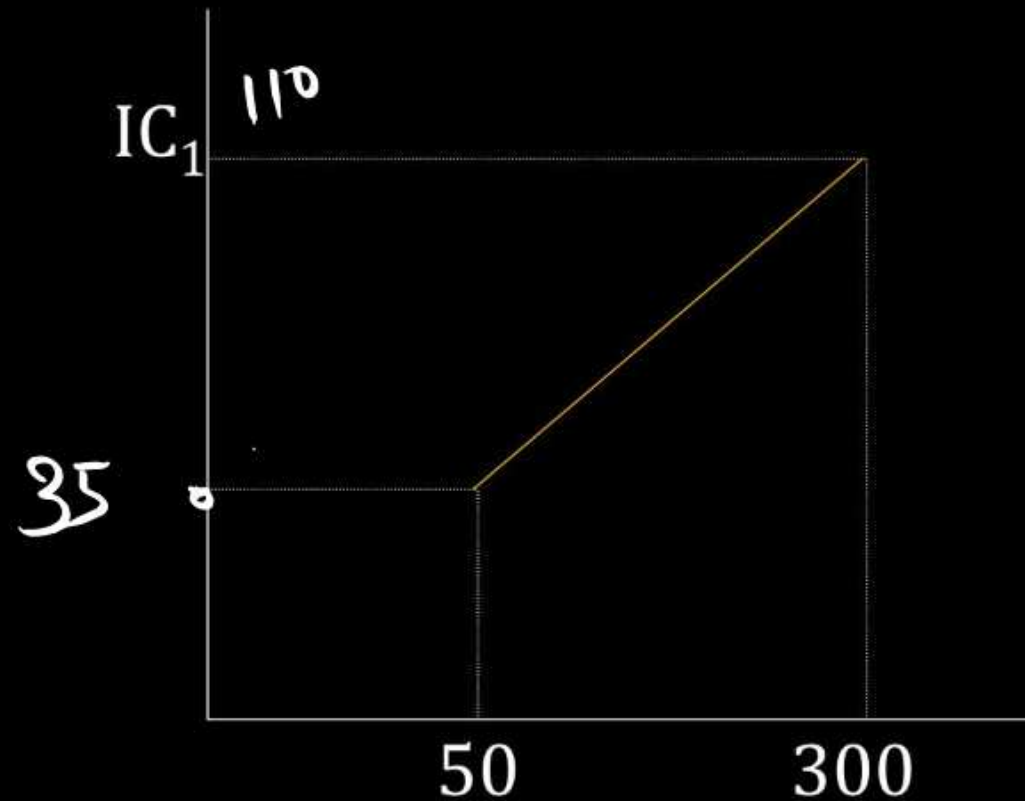
$$IC_1 = 20 + 0.3 P_1$$

$$IC_2 = 30 + 0.4 P_2$$

$$IC_3 = 30$$

If Minimum and maximum load on each unit are 50 MW & 300 MW respectively. If Total power demand is 700 MW. The power generated by each unit will be.

$$P_1 + P_2 + P_3 = 700$$



For Unit No 1.

$$\text{if } P=50 \Rightarrow IC = 20 + 0.3(50) = 35$$

$$P=300 \rightarrow IC = 20 + 0.3(300) = 110$$

For Unit No 2

$$\text{if } P=50 \Rightarrow IC = 30 + 0.4(50) = 50$$

$$P=300 \Rightarrow IC = 30 + 0.4(300) = 150$$

$$(Ic)_3 < [\text{Min}(Ic)_1 \text{ or } \text{Min}(Ic)_2]$$

$$P_3 = 300$$

$$P_1 + P_2 = 700 - 300$$

$$P_1 + P_2 = 400 \text{ --- (1)}$$

$$(Ic)_1 = (Ic)_2$$

$$20 + 0.3P_1 = 30 + 0.4P_2$$

$$0.3P_1 - 0.4P_2 = 10$$

$$3P_1 - 4P_2 = 100 \text{ --- (2)}$$

$$\begin{array}{rcl} \text{eqn 1} \times 4 & = & \\ 3P_1 - 4P_2 & = & 150 \\ 4P_1 + 4P_2 & = & 1650 \\ \hline \end{array}$$

$$7P_1 = 1750$$

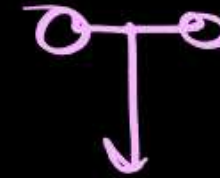
$$P_1 = 250$$

$$P_2 = 450 - 250 = 200$$

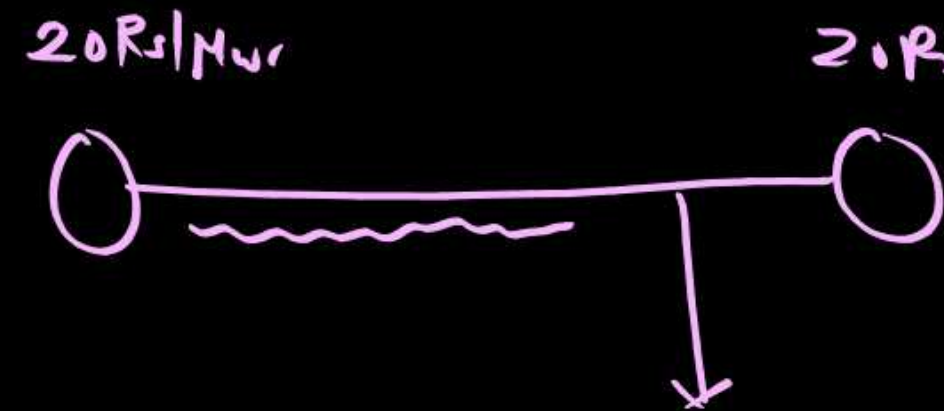
$$P_2 = 157.142$$

Optimum Load Dispatch Including Transmission Loss

⇒ If power station situated close to each other, then optimum scheduling can be done without T.L. loss.



⇒ However, if they are situated far from each other, scheduling must be done by taking T.L. losses into account.

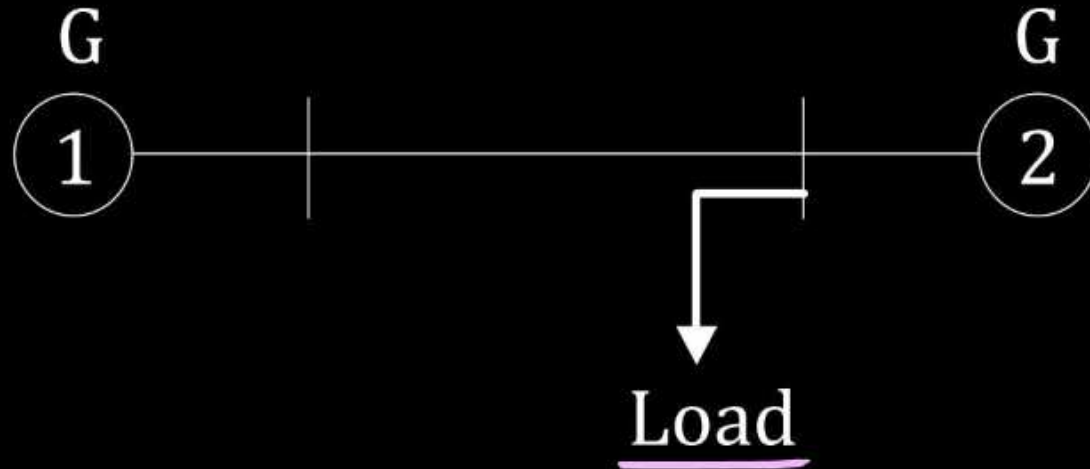


A young man and his father are sitting on a brown sofa in a living room. The young man, on the left, is wearing a blue t-shirt and jeans, and is gesturing with his hands while speaking. The father, on the right, is wearing a grey polo shirt and jeans, and is pointing his right index finger while responding. The background includes a window with sheer curtains, a potted plant on a side table, and a bookshelf filled with books.

Father, I will buy potatoes from Ramlal's shop because Ramlal provides us potatoes at a lower cost. No doubt.

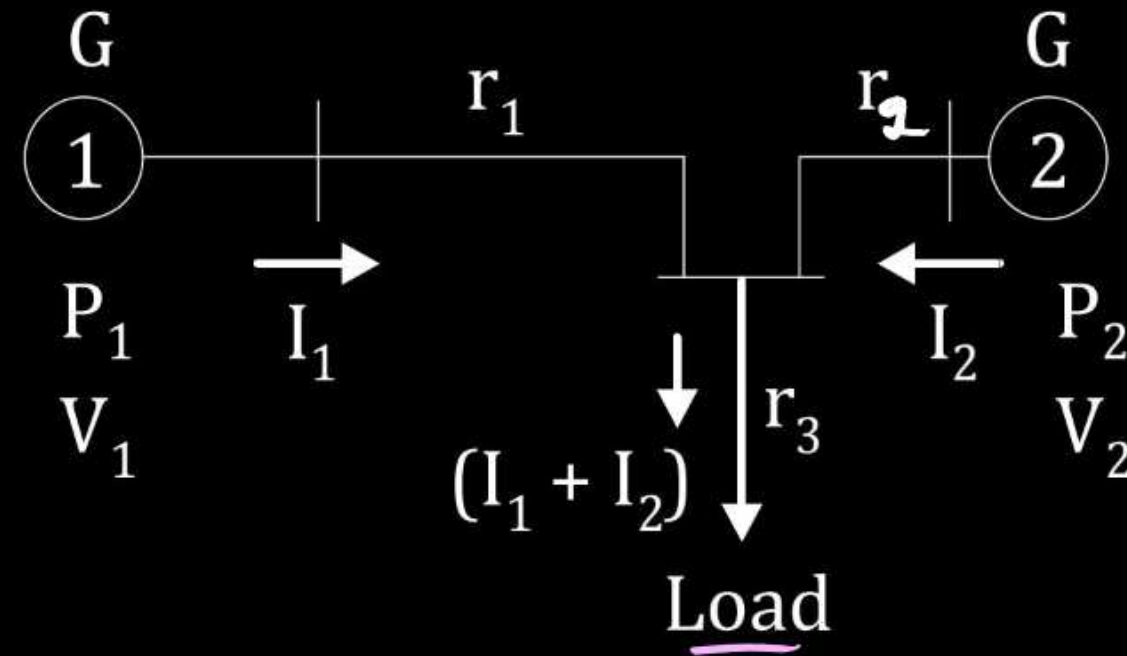
But Ramlal's shop is very far. You will also spend on petrol to go there. Whenever you consider cost, you should also consider the petrol cost. Only then you will know from where you get potatoes at the minimum cost.

Optimum Load Dispatch Including Transmission Loss



- ⇒ Suppose, if we have two units having same I.C. if we neglect T.L. losses than we should take half power from each unit.
- ⇒ But we take T.L. losses into account, we should take more power from unit 2 and less power from unit-1, so that losses will be Minimum.

Representation of Losses



Assume current I_1 & I_2 are in phase.

Total power loss in line

$$P_L = 3I_1^2 r_1 + 3I_2^2 r_2 + 3(I_1 + I_2)^2 r_3 = 3I_1^2 (r_1 + r_3) + 3I_2^2 (r_2 + r_3) + 6I_1 I_2 r_3 \quad - (i)$$

$$P_1 = \sqrt{3} V_1 I_1 \cos \phi_1$$

$$I_1 = \frac{P_1}{\sqrt{3} V_1 \cos \phi_1}$$

$$P_2 = \sqrt{3} V_2 I_2 \cos \phi_2$$

$$I_2 = \frac{P_2}{\sqrt{3} V_2 \cos \phi_2}$$

Put value of I_1 and I_2 in eqⁿ (1)

$$P_L = 3 \left(\frac{P_1}{\sqrt{3} V_1 \cos \phi_1} \right)^2 (\gamma_1 + \gamma_3) + 3 \left(\frac{P_2}{\sqrt{3} V_2 \cos \phi_2} \right)^2 (\gamma_2 + \gamma_3) + 6 \left(\frac{P_1}{\sqrt{3} V_1 \cos \phi_1} \right) \left(\frac{P_2}{\sqrt{3} V_2 \cos \phi_2} \right) \gamma_3$$

$$P_L = \underbrace{\frac{(\gamma_1 + \gamma_3)}{(V_1 \cos \phi_1)^2}}_{B_{11}} P_1^2 + \underbrace{\frac{(\gamma_2 + \gamma_3)}{(V_2 \cos \phi_2)^2}}_{B_{22}} P_2^2 + \underbrace{\frac{2 \gamma_3}{(V_1 \cos \phi_1)(V_2 \cos \phi_2)}}_{B_{12}} P_1 P_2$$

$$P_L = B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2$$

$$B_{11} = \frac{\gamma_1 + \gamma_3}{(V_1 \cos \phi_1)^2}$$

$$B_{22} = \frac{\gamma_2 + \gamma_3}{(V_2 \cos \phi_2)^2}$$

$$B_{12} = \frac{\gamma_3}{(V_1 \cos \phi_1)(V_2 \cos \phi_2)}$$

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + 2P_1 P_2 B_{12}$$

- B_{11} , B_{22} , B_{12} ... etc are called loss coefficient.
- If all quantities are in pu then these coefficient will also be in pu.
- If voltage are line to line voltage & line resistance are in ohm. The unit of this will be (MW)⁻¹.
- B depend upon voltage & power factor, so it also vary with system operating condition. But if load variation are small, we assume it as a constant for some average operating condition.

→ → The general form of loss equation.

$$P_L = \sum_n \sum_m P_m B_{mn} P_n$$

$$\begin{aligned} \text{E.g. } P_L &= P_1 [P_1 B_{11} + P_2 B_{21}] + P_2 [P_1 B_{11} + P_2 B_{22}] \\ &= P_1^2 B_{11} + P_2^2 B_{22} + 2P_1 P_2 B_{12} \end{aligned}$$

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + P_3^2 B_{33} + 2P_1 P_2 B_{12} + 2P_2 P_3 B_{23} + 2P_3 P_1 B_{13}$$

or In Matrix form.

$$P_L = P^T B P$$

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_k \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1k} \\ B_{21} & B_{22} & \cdots & B_{2k} \\ \vdots & \cdot & \cdot & \cdot \\ B_{k1} & B_{k2} & \cdots & B_{kk} \end{bmatrix}$$

For two Bus System

$$\begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

- In 2 Bus system it has 4 loss coefficient $B_{11}, B_{22}, B_{12} \text{ \& } B_{21}$. $B_{12} = B_{21}$ 
- So In 3 Bus system it has $(3)^2 = 9$ loss coefficient. $B_{12} = B_{21}$ $B_{23} = B_{32}$
- In N Bus system it has $(N)^2$ loss coefficient.
- As size of power system increases it is so design (Interconnected) so that most of loss coefficient be become zero so Reduce power loss.
- B_{ii} or B_{jj} is called self loss coefficient.
- The B_{ij} is called Mutual loss coefficient.
- In large Interconnected system we can Make Mutual loss coefficient equal to zero, but self loss coefficient are generally not zero.



2 Minutes Summary

→ ELD

→ Different Cases

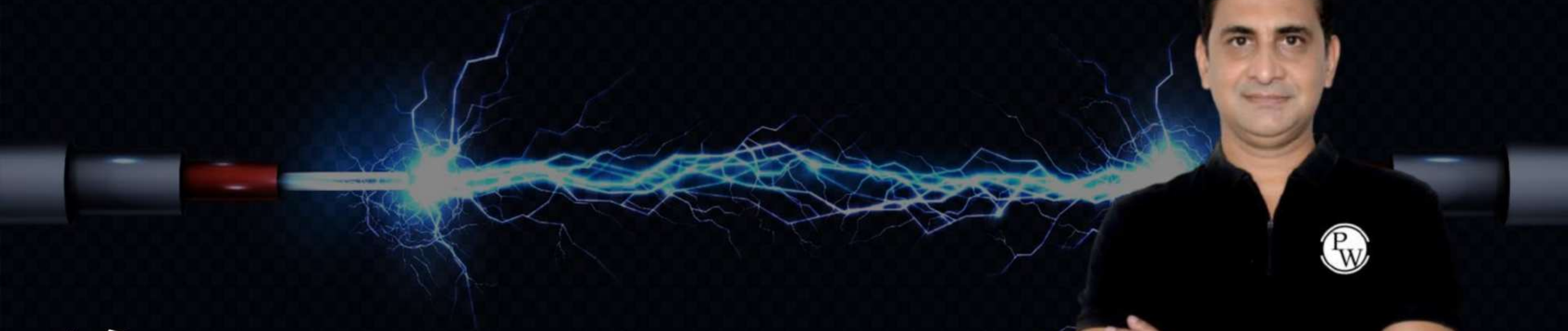
→ loss coefficient

Thank
you

Electrical Engineering



Power System



Lecture No 2 Economic Load Dispatch



By- Ajay Gupta Sir



Recap of Previous class

1. Different cases of ELD

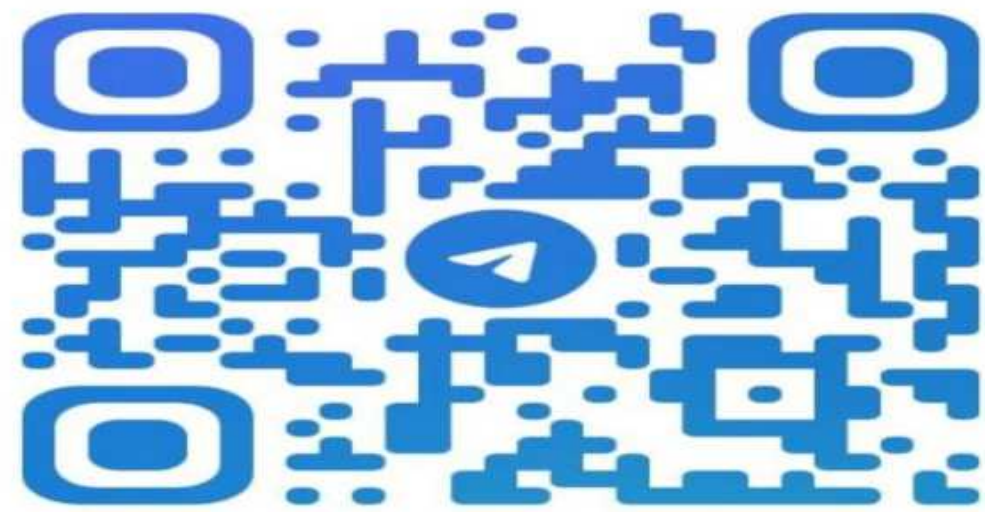
2..

3. Loss Coefficient.



Topics to be Covered

- ✓ 1. Droop Characteristic
- ✓ 2. Action of governor
- ✓ 3. Problems.
- ✓ 4. ELD with Losses



@AJAYGUPTASIRPW

Lagrangian Method (Including loss)

- Suppose n unit are giving $P_1, P_2, \dots, P_n$ power and cost of power are $F_1, F_2, \dots, F_n$ respectively.

- Total power generation = Demand + Losses



- $P_T = P_D + P_L$ $P_D + P_L = \sum_{n=1}^n P_n$

- $P_1 + P_2 + \dots + P_n = P_D + P_L$

Lagrangian Method (Including loss)

- So, constraint equation

$$P_D + P_L - \Sigma P_n = 0$$

- Motive is to reduce Total power cost.

$$F_T = \underline{F_1 + F_2 + F_3 \dots + F_n} \text{ (Main function)}$$

- Auxiliary function = Main function + λ (constraint)

$$\underline{F} = \underline{F_T} + \lambda(\underline{P_D + P_L - \Sigma P_n})$$

$$F = F_1 + F_2 + \dots + F_n + \lambda [P_D + P_L - (P_1 + P_2 + \dots + P_n)]$$

$$\rightarrow \frac{dF}{dP_1} = 0$$

$$\frac{dF_1}{dP_1} + \frac{dF_2}{dP_1} + \dots + \frac{dF_n}{dP_1} + \lambda \left[0 + \frac{\partial P_L}{\partial P_1} - 1 \right]$$

$$\frac{dF_1}{dP_1} + 0 = \lambda \left[1 - \frac{\partial P_L}{\partial P_1} \right]$$

$$\lambda = \frac{dF_1}{dP_1} \cdot \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \quad \text{--- ①}$$

$$\rightarrow \frac{dF}{dP_2} = 0$$

$$\frac{dF_1}{dP_2} + \frac{dF_2}{dP_2} + \dots + \frac{dF_n}{dP_2} + \lambda \left[0 + \frac{\partial P_L}{\partial P_2} - 1 \right]$$

$$\frac{dF_2}{dP_2} = \lambda \left[1 - \frac{\partial P_L}{\partial P_2} \right]$$

$$\lambda = \frac{dF_2}{dP_2} \cdot \frac{1}{\left[1 - \frac{\partial P_L}{\partial P_2} \right]} \quad - (2)$$

Similarly

$$\lambda = \frac{dF_n}{dP_n} \cdot \frac{1}{1 - \frac{\partial P_L}{\partial P_n}} \quad - (n)$$

So from eqn ① ② ... ⑦

$$\frac{dF_1}{dP_1} \cdot \frac{1}{\left(1 - \frac{\partial P_L}{\partial P_1}\right)} = \frac{dF_2}{dP_2} \cdot \frac{1}{\left(1 - \frac{\partial P_L}{\partial P_2}\right)} = \dots = \frac{dF_n}{dP_n} \cdot \frac{1}{\left(1 - \frac{\partial P_L}{\partial P_n}\right)} = \lambda$$

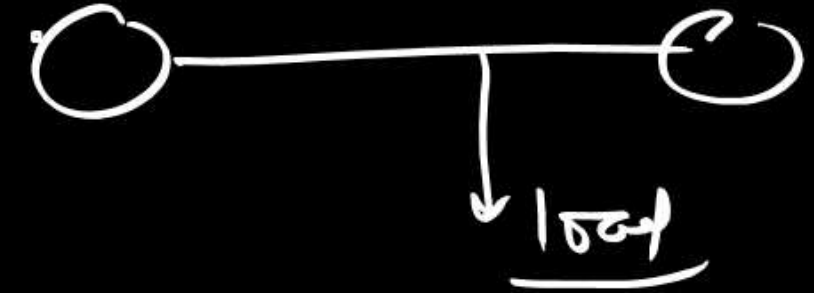
$$\frac{dF_1}{dP_1} \cdot L_1 = \frac{dF_2}{dP_2} \cdot L_2 + \dots = \frac{dF_n}{dP_n} \cdot L_n = \lambda$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \quad \text{Penalty factor for unit 1}$$

L_n = penalty factor of n unit. it is a unit less quantity.



$$\frac{dF_1}{dP_1} L_1 = \frac{dF_2}{dP_2} L_2 \dots = \frac{dF_n}{dP_n} L_n = \lambda$$



So, for optimal scheduling. The product of penalty factor & I.C. should be equal to lagrangian Multiplier of system.

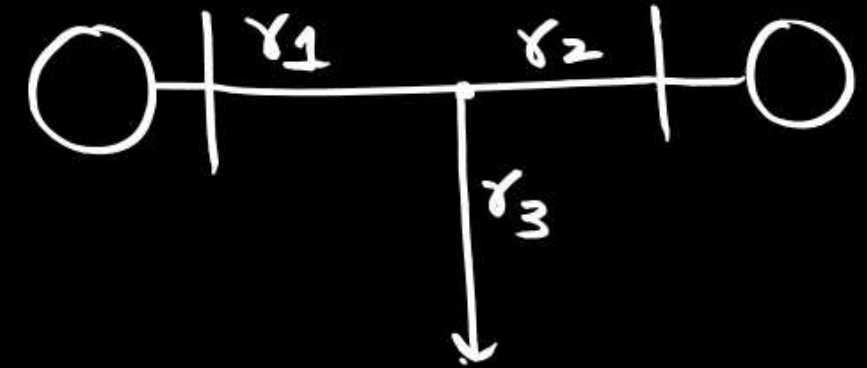
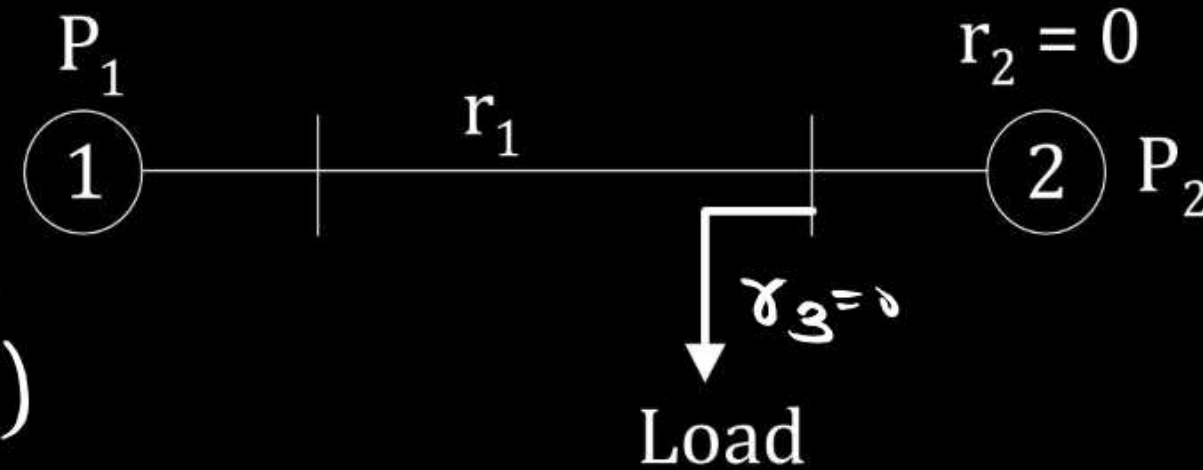
$$(IC)_1 L_1 = (IC)_2 L_2 = \dots = (IC)_n L_n = \lambda$$

λ = Incremental Cost at load end.

Special Cases

$$B_{22} = \frac{\gamma_2 + \gamma_3}{(V_2 \cos \phi_2)^2}$$

$$B_{12} = \frac{\gamma_3}{(V_1 \cos \phi_1)(V_2 \cos \phi_2)}$$



- If total load is connected at Bus-2 (near to station 2). The losses supplied by station 2 will be zero, so loss coefficient connected with station 2 will always zero.

$$P_L = B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2$$

$$\checkmark B_{12} = 0 \quad \checkmark B_{22} = 0$$

$$P_L = B_{11} P_1^2$$

$$\frac{\partial P_L}{\partial P_1} = \underbrace{2B_{11} P_1} \neq 0, \quad L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \neq 1$$

$$\frac{\partial P_L}{\partial P_2} = 0, \quad \underline{L_2} = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \underline{1}$$

Similarly if total load is connected at unit 1.

$$B_{\underline{11}} = 0 \quad B_{\underline{12}} = 0$$

$$P_L = \underline{B_{11}} P_1^2 + B_{22} P_2^2 + 2 P_1 P_2 \underline{B_{12}}$$

$$P_L = B_{\underline{22}} P_2^2$$

$$\frac{\partial P_L}{\partial P_2} = 2 B_{22} P_2, \quad \underline{L_2} = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} \neq 1$$

$$\frac{\partial P_L}{\partial P_1} = 0, \quad \underline{L_1} = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = 1$$

Q.



The power system has three generating plant and power is dispatched economically with $P_1=150$, $P_2=100$ and $P_3=200$. The loss coefficient are. $B_{11}=0.0010$, $B_{22}=0.0015$, $B_{33}=0.0020$, $B_{12}=-0.0001$, $B_{23}=-0.0002$ and $B_{13}=-0.0004$. Find out the penalty factor for plant-1

$$P_L = P^T B T$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$$

$$P_L = \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

- A 1.00
- B 2.00
- C 1.13
- D None

$$P_L = B_{11} P_1^2 + B_{22} P_2^2 + B_{33} P_3^2 + 2B_{12} P_1 P_2 + 2B_{23} P_2 P_3 + 2B_{13} P_1 P_3$$

$$\frac{\partial P_L}{\partial P_1} = 2B_{11} P_1 + 2B_{12} P_2 + 2B_{13} P_3$$

$$= 2(0.0010)(150) + 2(-0.0001)(100) + 2(-0.0004)(200)$$

$$= 0.12$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.12} = 1.136$$

Q.

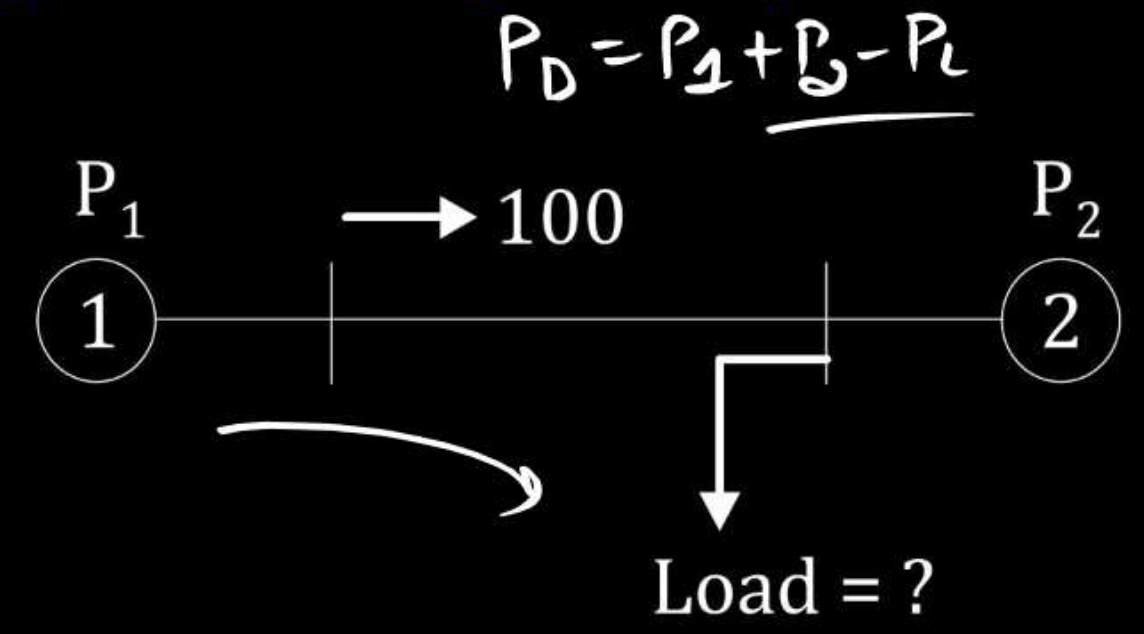


A system consist of two plant connected by T.L. The load is at plant 2. The T.L. losses calculated revealed that a transfer of 100 MW power from plant 1 to 2 Means a losses of 15 MW. Find the required generation by each plant for $\lambda = 60$.

$$\frac{dC_1}{dP_1} = 0.2P_1 + 22 \text{ Rs | MWh}$$

$$\frac{dC_2}{dP_2} = 0.15P_2 + 30 \text{ Rs | MWh}$$

Total load (Demand) = ?



- A 100
- B 200
- C 115
- D 285 ✓

$$B_{22} = 0 \quad B_{12} = 0$$
$$P_L = B_{11} P_1^2$$

if $P_1 = 100$
 $P_2 = 15$

$$\text{if } P_1 = 100 \Rightarrow P_L = 15 \text{ MW}$$

$$15 = B_{11} (100)^2$$

$$B_{11} = \frac{15}{10000} = 0.0015$$

$$P_L = B_{11} P_1^2$$

$$\rightarrow P_L = 0.0015 P_1^2$$

$$\frac{\partial P_L}{\partial P_1} = 0.003 P_1$$

$$\rightarrow L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.003 P_1}$$

$$\frac{\partial P_L}{\partial P_2} = 0$$

$$\rightarrow L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \frac{1}{1 - 0} = 1$$

$$(Ic)_1 L_1 = \lambda$$

$$(0.2P_1 + 22) \frac{1}{1 - 0.003P_1} = 60$$

$$0.2P_1 + 22 = 60 - 0.18P_1$$

$$0.38P_1 = 38$$

$$P_1 = 100$$

$$(Ic)_2 L_2 = \lambda$$

$$(0.15P_2 + 30) \cdot 1 = 60$$

$$0.15P_2 = 30$$

$$P_2 = 200$$

$$\rightarrow \text{if } P_1 = 100 \Rightarrow P_L = 15$$

$$P_D = P_1 + P_2 - P_L$$

$$P_D = 100 + 200 - 15 = 285$$

Q.



The figure shows a two-generator system supplying a load of $P_D = 40$ MW, connected at bus 2.

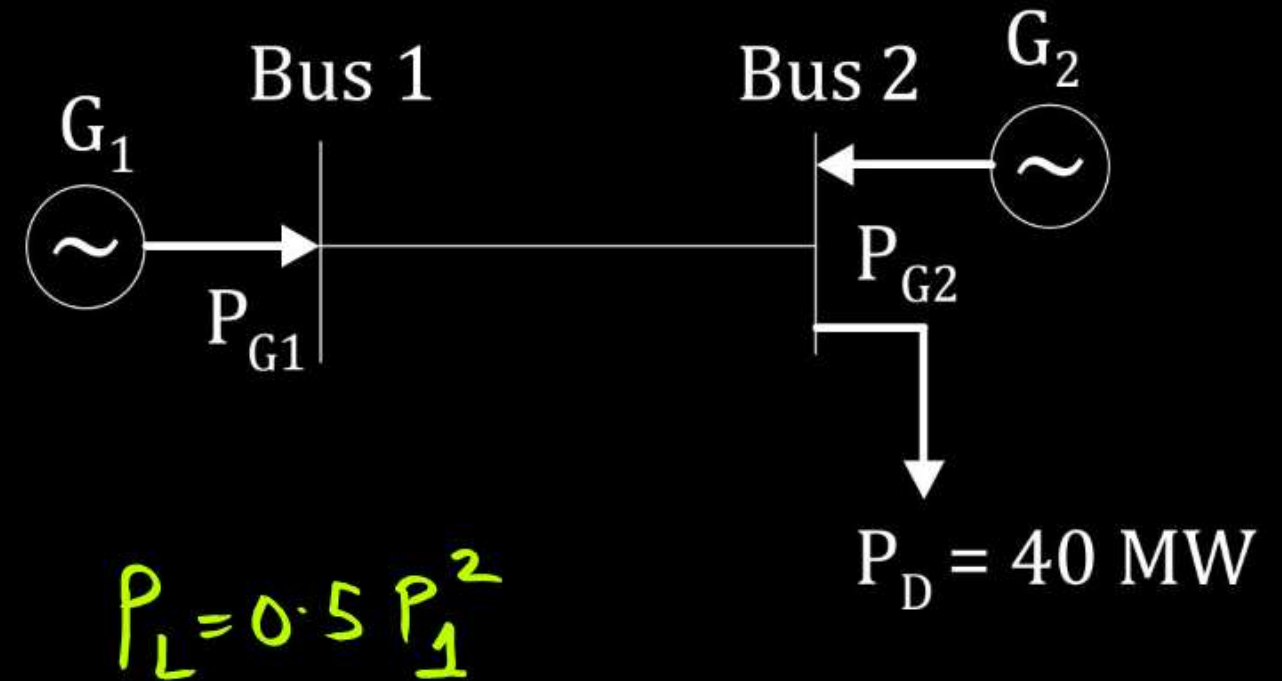
The fuel cost of generators G_1 and G_2 are:

$$C_1(P_{G1}) = 10,000 \text{ Rs/MWh and}$$

$$C_2(P_{G2}) = 12,500 \text{ Rs/MWh}$$

and the loss in the line is $P_{\text{loss(pu)}} = 0.5 P_{G1(\text{pu})}^2$, where the loss coefficient is specified in pu on a 100 MVA base.

The most economic power generation schedule in MW is



✓
A $P_{G1} = 20, P_{G2} = 22$

B $P_{G1} = 22, P_{G2} = 20$

C $P_{G1} = 20, P_{G2} = 20$

D $P_{G1} = 0, P_{G2} = 40$

$$P_L = 0.5 P_1^2$$

$$\frac{\partial P_L}{\partial P_1} = P_1$$

$$\rightarrow L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - P_1}$$

$$\frac{\partial P_L}{\partial P_2} = 0$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = 1$$

$$-(IC)_1 L_1 = (IC)_2 L_2$$

$$10,000 \cdot \frac{1}{1 - P_1} = 12500 \cdot 1$$

$$100 = 125 - 125 P_1$$

$$125 P_1 = 25$$

$$P_1 = \frac{1}{5} = 0.2 \text{ p.u.}$$

$$\begin{aligned} \rightarrow P_L &= 0.5 P_1^2 \\ &= 0.5 (0.2)^2 = 0.02 \text{ p.u.} \end{aligned}$$



$$P_1(\text{Actual}) = 0.2 \times 100 = 20 \text{ MW}$$

$$P_L(\text{Actual}) = 0.02 \times 100 = 2 \text{ MW}$$

$$P_1 + P_2 = P_D + P_L$$

$$20 + P_2 = 40 + 2$$

$$P_2 = 22$$

Q.

The fuel inputs of plants 1 and 2 are given as

$$F_1 = 0.2P_1^2 + 40P_1 + 120 \text{ Rs/hour}$$

$$F_2 = 0.25P_2^2 + 30P_2 + 150 \text{ Rs/hour}$$

Determine the economic operating schedule and the corresponding cost of generation, if the maximum and minimum loading of each machine is 100 MW and 25 MW, the demand is 180 MW and transmission losses are neglected. If the load is equally shared by both units, determine the saving obtained by loading the units as per incremental cost.

Q.

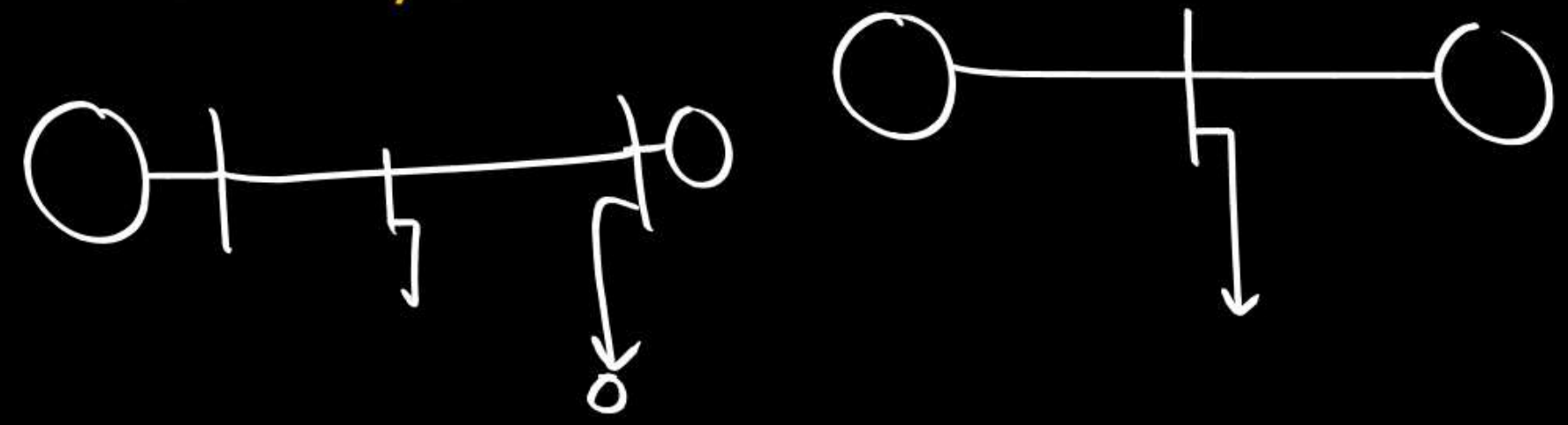


The method of getting the solution to an optimization problem with neglected transmission losses:

1. Does sense the changes in the load. ✓
2. Does not sense the location of the change in load. ✓
3. Does sense the changes in the load and the location of changes in the loads.
4. Does not sense both the location and the changes in the load.

Which of the above statements is/are true

- A 1 and 2 only ✓
- B Either 3 or 4
- C 4 only
- D 3 only



MCQ

$$(I_c)_1 = (I_c)_2$$



The expressions of fuel cost of two thermal generating units as a function of the respective power generation P_{G1} and P_{G2} are given

as $F_1(P_{G1}) = 0.1aP_{G1}^2 + 40P_{G1} + 120 \text{ Rs/hour} \quad 0 \text{ MW} \leq P_{G1} \leq 350 \text{ MW}$

$$F_2(P_{G2}) = 0.2P_{G2}^2 + 30P_{G2} + 100 \text{ Rs/hour} \quad 0 \text{ MW} \leq P_{G2} \leq 300 \text{ MW}$$

where a is a constant. For a given value of a , optimal dispatch requires the total load of 290 MW to be shared as $P_{G1} = \underline{175 \text{ MW}}$ and $P_{G2} = \underline{115 \text{ MW}}$. With the load remaining unchanged, the value of a is increased by 10% and optimal dispatch is carried out. The changes in P_{G1} and the total cost of generation, $F(= F_1 + F_2)$ in Rs hour will be as follows

[GATE-2023: 1M]

$$(IC)_1 = (IC)_2$$

$$0.2aP_1 + 40 = 0.4P_2 + 30$$

$$0.2a(175) + 40 = 0.4(115) + 30$$

$$35a + 40 = 76$$

$$a = 1.0285$$

→ if a is increase by 1%.

$$\begin{aligned} a(\text{new}) &= 1.028 \times 1.1 \\ &= 1.1314 \end{aligned}$$

$$0.2(1.1314)P_1 + 40 = 0.4P_2 + 30$$

$$0.2262P_1 - 0.4P_2 = -10 \quad \text{--- (1)}$$

$$P_1 + P_2 = 290 \quad \text{--- (2)}$$

$$P_2 = 290 - P_1$$

Put this in eqⁿ (1)

$$0.2262P_1 - 0.4(290 - P_1) = -10$$

$$0.6262P_1 = 106$$

$$P_1 = 169.275 \quad \text{--- } P_1 \text{ is do}$$

$$P_2 = 290 - 169.275$$

$$P_2 = 120.725$$

$$F = F_1 + F_2 \quad (P_1 = 175 \quad P_2 = 115)$$

$$F = 0.1(1.0285)P_1^2 + 40P_1 + 120 + 0.2P_2^2 + 30P_2 + 180$$

$$F = 0.10285(175)^2 + 40(175) + 120 + 0.2(115)^2 + 30(115) + 180$$

$$F = 16464.78$$

$$F(\text{new}) = F_1 + F_2$$

$$= 0.1(1.1314)P_1^2 + 40P_1 + 120 + 0.2P_2^2 + 30P_2 + 180$$

$$= 0.11314(169.275)^2 + 40(169.275) + 120 + 0.2(120.725)^2 + 30(120.725) + 180 = 16769.57 \quad (\text{Increase})$$

- A** P_{G1} will decrease and F will increase
- B** Both P_{G1} and F will increase
- C** P_{G1} will increase and F will decrease
- D** Both P_{G1} and F will decrease



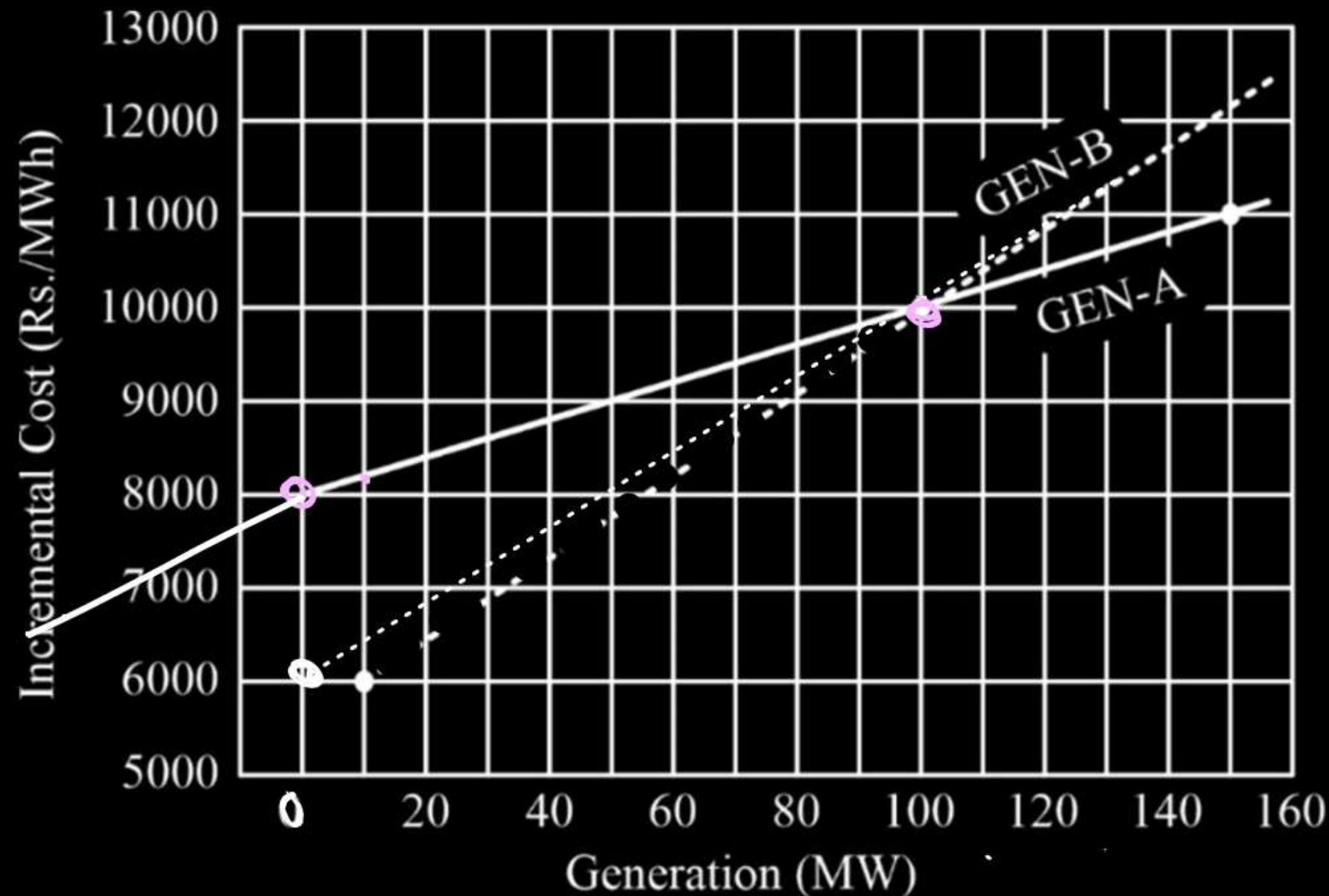
#Q. The incremental cost curves of two generators (Gen A and Gen B) in a plant supplying a common load are shown in the figure. If the incremental cost of supplying the common load is Rs. 7400 per MWh, then the common load in MW is _____ (rounded off to the nearest integer).

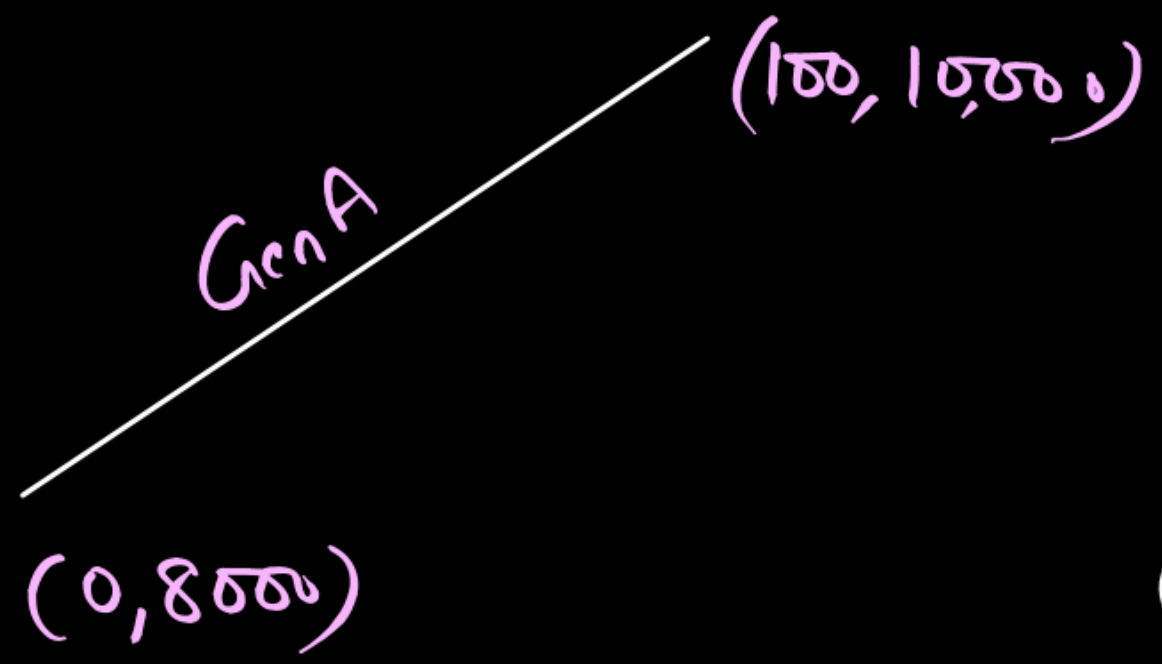
$$(IC)_1 = 7400$$

$$P_1 = ?$$

$$(IC)_2 = 7400$$

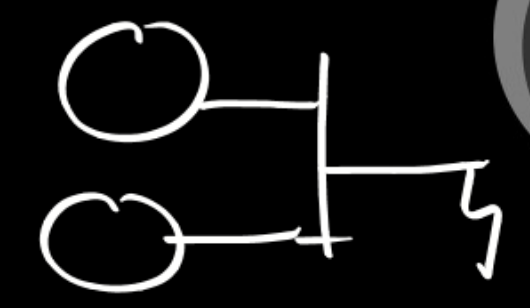
$$P_2 = ?$$





$$y = Ic$$

$$x = P$$



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

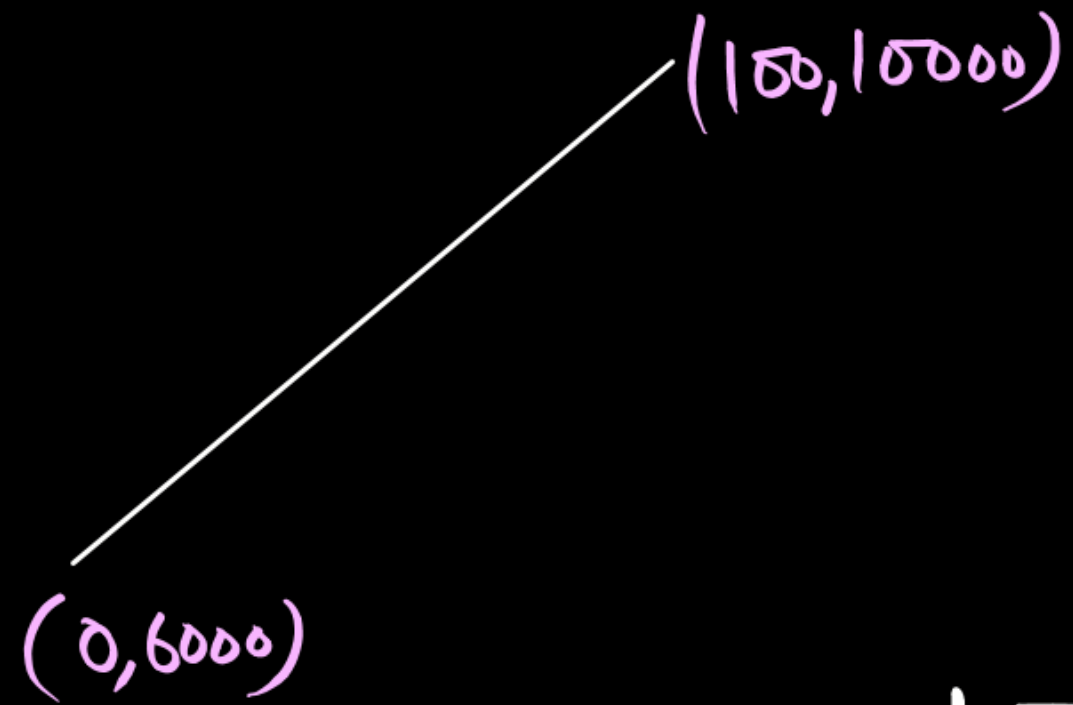
$$(Ic)_A - 8000 = \frac{10,000 - 8000}{100 - 0} (P_A - 0)$$

$$(Ic)_A - 8000 = 20 P_A$$

$$\therefore (Ic)_A = 7400$$

$$7400 - 8000 = 20 P_A$$

$$P_A = -30 \text{ MW} \quad \left[\text{So A Generator does not give power} \right]$$



$$(IC)_B - 6000 = \frac{10,000 - 6000}{100 - 0} (P_B - 0)$$

$$(IC)_B - 6000 = 40 P_B$$

$$\text{if } IC = 7400$$

$$7400 - 6000 = 40 P_B$$

$$P_B = 35$$

$$\text{Total load} = P_A + P_B = 0 + 35 = \underline{35}$$

[NAT]

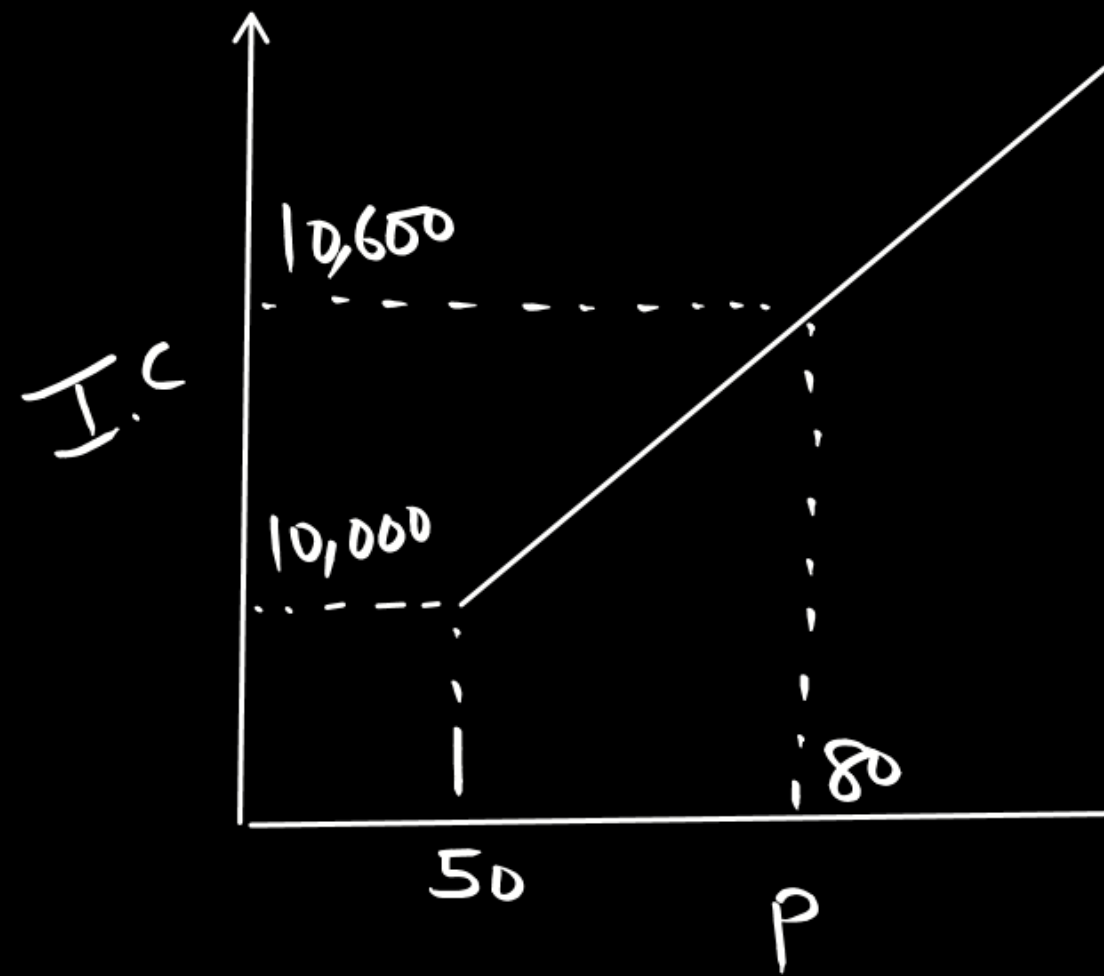


#Q. Two units, rated at 100 MW and 150 MW, are enabled for economic load dispatch. When the overall incremental cost is 10,000 Rs./MWh, the units are dispatched to 50 MW and 80 MW respectively. At an overall incremental cost of 10,600 Rs./MWh, the power output of the units are 80 MW and 92 MW, respectively. The total plant MW-output (without overloading any unit) at an overall incremental cost of 11,800 Rs./MWh is ____ (round off to the nearest integer).

[2 Marks]

$$\begin{cases} \text{If } IC = 10,000 & P_1 = 50 & P_2 = 80 \\ IC = 10,600 & P_1 = 80 & P_2 = 92 \end{cases}$$

$$\text{If } IC = 11,800 \quad = P_1 = ? \quad P_2 = ?$$



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$(IC)_1 - 10,000 = \frac{10,600 - 10,000}{80 - 50} (P_1 - 50)$$

$$(IC)_1 - 10,000 = 20(P_1 - 50)$$

$$\text{if } IC = 11,800$$

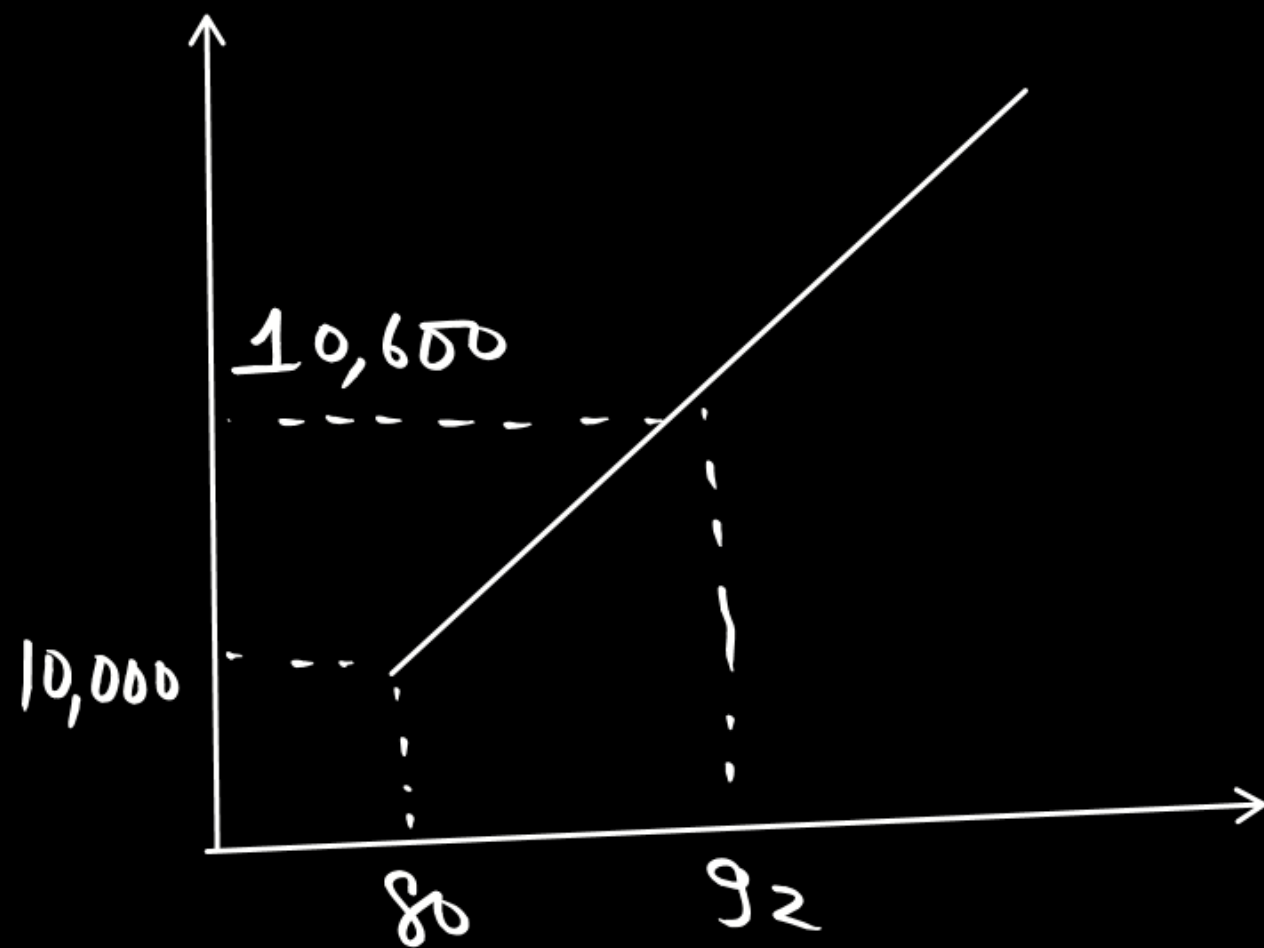
$$11,800 - 10,000 = 20(P_1 - 50)$$

$$P_1 - 50 = 30$$

$$P_1 = 140$$

$$P_1 > P_2 \text{ (Rating)}$$

$$P_1 = 100 \text{ Mw}$$



$$(IC)_2 - 10,000 = \frac{10,600 - 10,000}{92 - 80} (P_2 - 80)$$

$$(IC)_2 - 10,000 = 50 (P_2 - 80)$$

$$\text{if } IC = 11,800$$

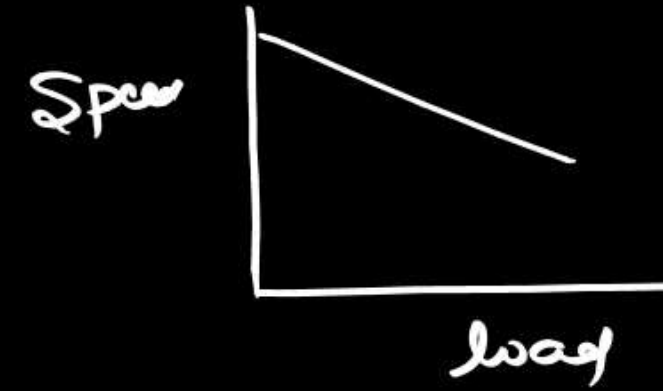
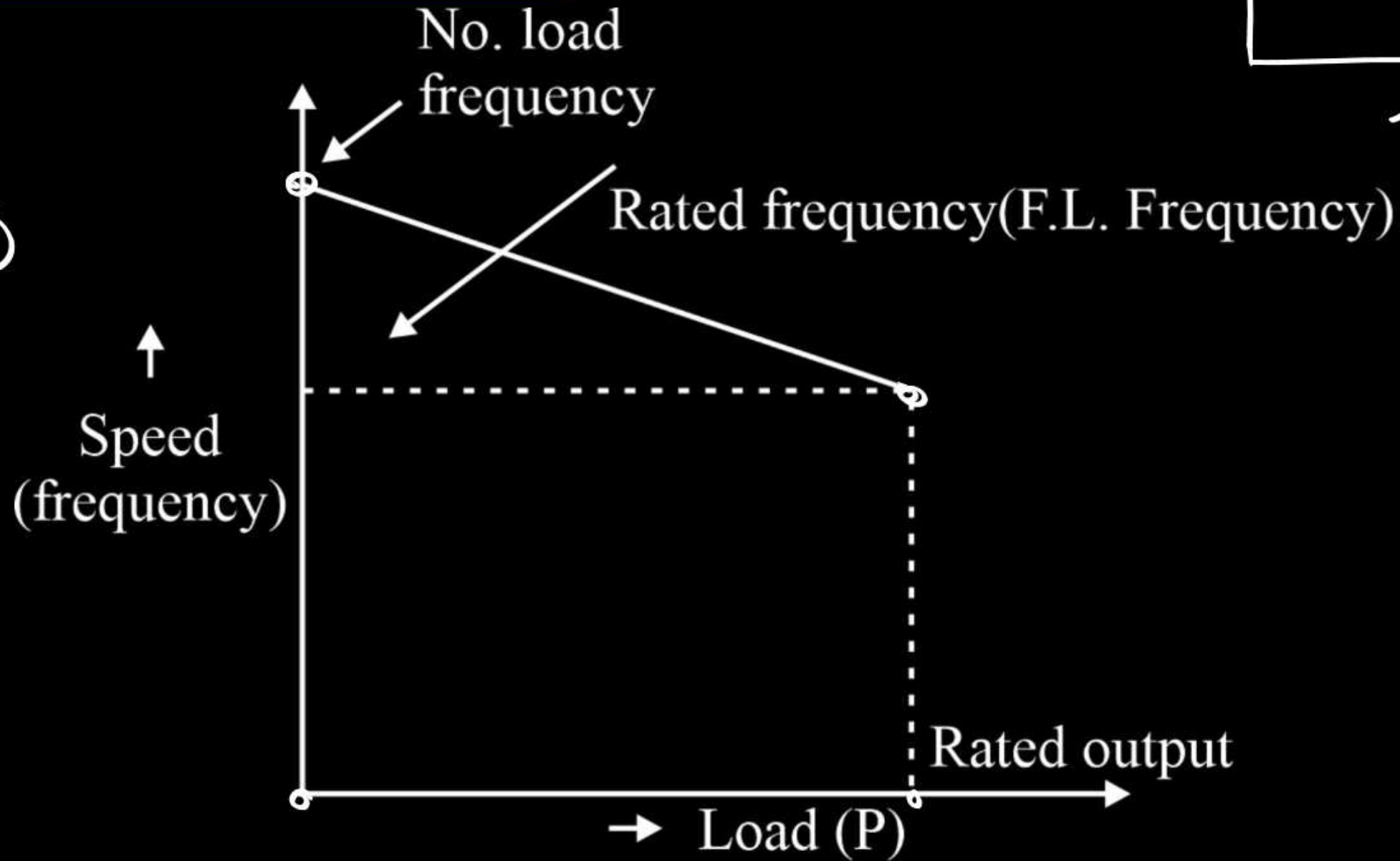
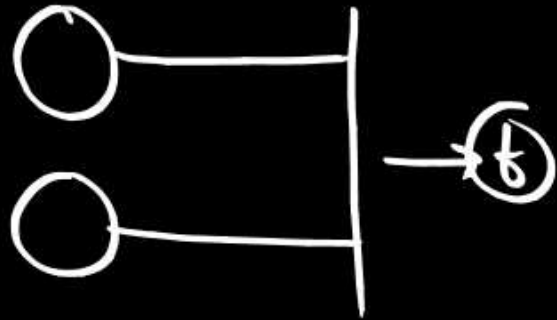
$$11,800 - 10,000 = 50 (P_2 - 80)$$

$$P_2 = 116$$

$$\begin{aligned}\text{Total} &= P_1 + P_2 \\ &= 100 + 116 \\ &= \underline{216}\end{aligned}$$



Droop Characteristic

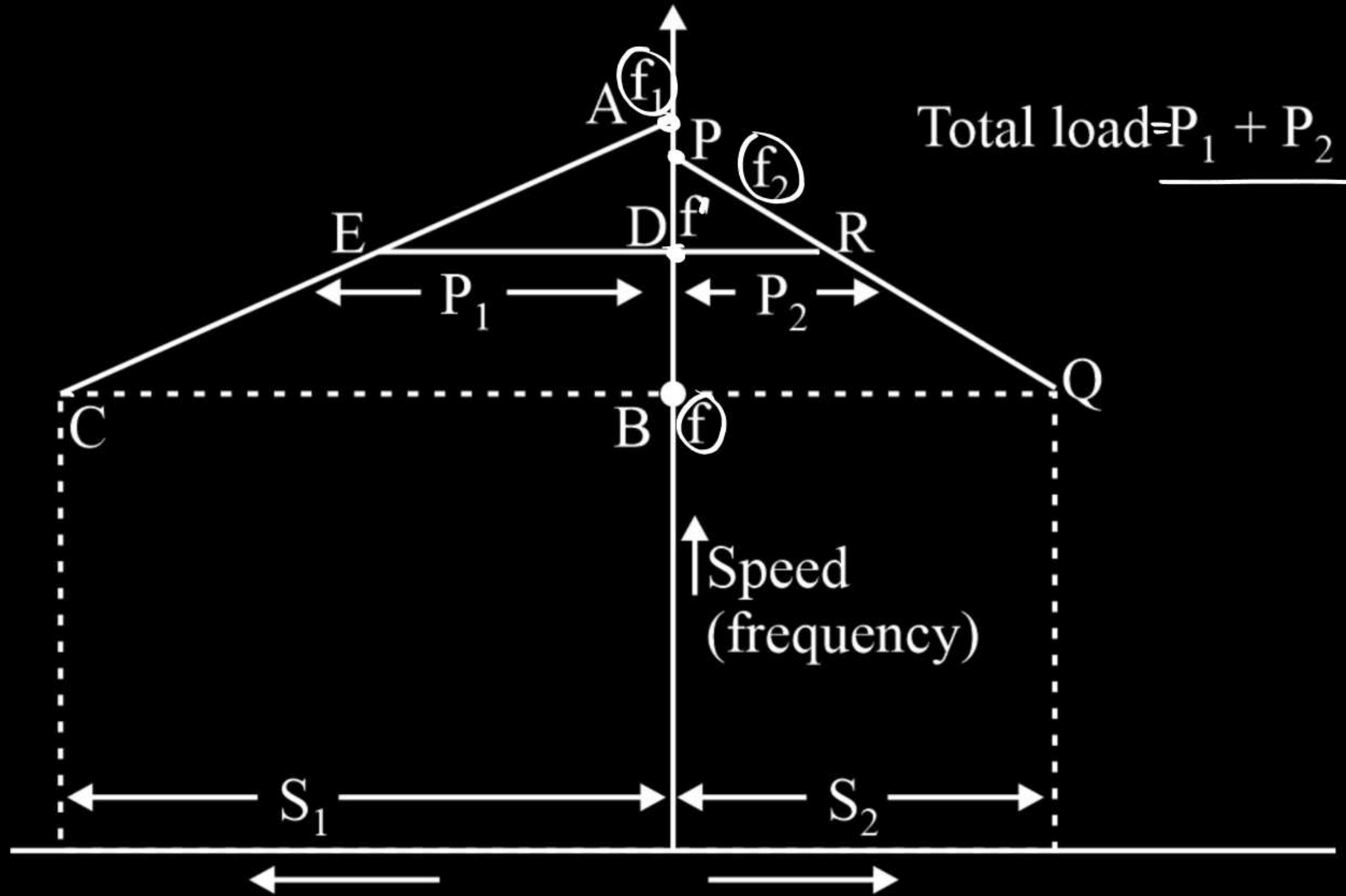


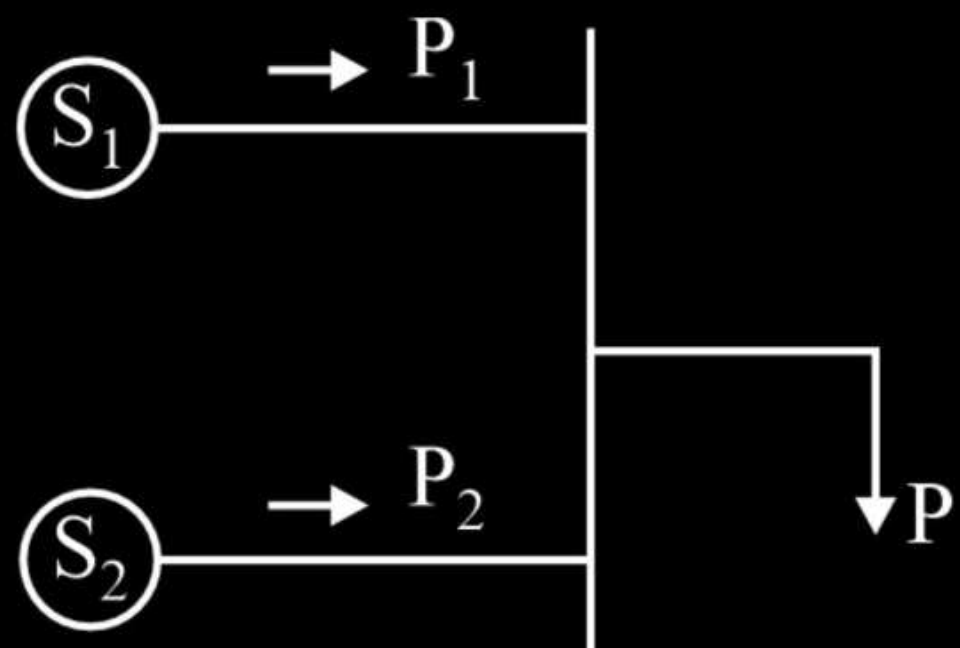
$$\phi = \frac{P\pi}{120}$$

In any alternator, if load is increase, its speed decreases, The characteristic between load and speed in called droop characteristic.



Parallel operation of two Alternator







No load frequency of alternator 1 = f_1

No load frequency of alternator 2 = f_2

F.L. Frequency = f

Rating of Alternator 1 = S_1

Rating of Alternator 2 = S_2

The load supplied by alternator 1 = P_1 ✓

The load supplied by alternator 2 = P_2 ✓

And frequency at $P = P_1 + P_2$ load = f'

$\triangle AED$ & $\triangle ACB$ is similar triangle

$$\frac{ED}{BC} = \frac{AD}{AB}$$

$$\frac{P_1}{S_1} = \frac{t_1 - t'}{t_1 - t}$$

$$P_1 = S_1 \left(\frac{t_1 - t'}{t_1 - t} \right) \quad \text{--- (1)}$$

$\triangle PDR$ and $\triangle PDO$ Similar

$$\frac{DR}{BQ} = \frac{PD}{PB}$$

$$\frac{P_2}{S_2} = \frac{t_2 - t'}{t_2 - t}$$

$$P_2 = S_2 \left[\frac{t_2 - t'}{t_2 - t} \right] \quad \text{--- (2)}$$

$$P_1 + P_2 = P_D$$

$$S_1 \left(\frac{t_1 - t'}{t_1 - t} \right) + S_2 \left(\frac{t_2 - t'}{t_2 - t} \right) = P_D$$

Now find out t'

Now using this t' , find out P_1 and P_2

#Q. Two similar alternators operating in parallel have the following data:

Alternator 1- Capacity 700 kW, frequency drops from 50 Hz at no load to 48.5 Hz at full load.
~~50~~

Alternator 2- Capacity 700 kW, frequency drops from 50.5 Hz at no load to 48 Hz at full load.

Speed regulation of prime-movers – linear in each case.

(a) Calculate how a total load of 1200 kW is shared by each alternator. Also find the operating bus-bar frequency at this load.

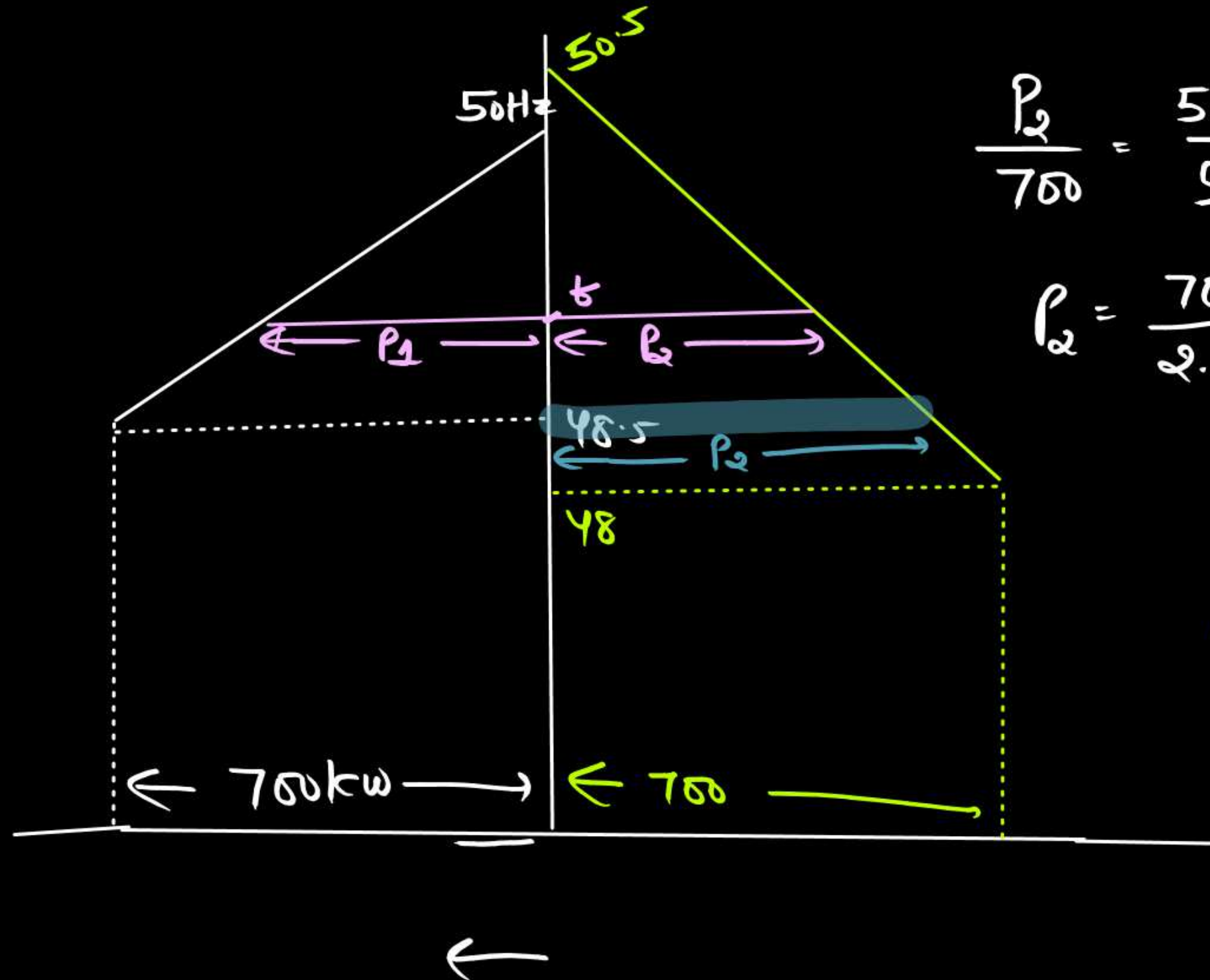
(b) Compute the maximum load that these two units can deliver without overloading either of them.

$$\frac{P_1}{700} = \frac{50 - t}{50 - 48.5}$$

$$P_1 = \frac{700(50 - t)}{1.5}$$

$$\frac{P_2}{700} = \frac{50.5 - t}{50.5 - 48}$$

$$P_2 = \frac{700(50.5 - t)}{2.5}$$



$$P_1 + P_2 = 1200$$

$$\frac{700}{1.5} (50 - f) + \frac{700}{2.5} (50.5 - f) = 1200$$

$$f = 48.58 \text{ Hz.}$$

$$P_1 = \frac{700}{1.5} (50 - 48.58) = 662.67$$

$$P_2 = \frac{700}{2.5} (50.5 - 48.58) = 537.6$$

→ without overloading any of the generator. Generator 1 is fully loaded 700 kW

$$\frac{P_2}{700} = \frac{50.5 - 48.5}{50.5 - 48}$$

$$P_2 = 560$$

$$\begin{aligned} P(\text{Max}) &= P_1 + P_2 \\ &= 700 + 560 \\ &= \underline{1260} \end{aligned}$$



Action of Governor:

Two alternators are running in parallel supplying total load of P at the frequency of f . The alternators 1 is supplying P_1 power and alternators 2 is supplying P_2 power.

If by governor setting, if the prime mover torque of alternator 1 is increased, speed will be increased, and droop characteristic will shift upward direction from position 1 to 2.

Now the operating frequency will increase from f to f' and load supplied by each generator is become P'_1 and P'_2 .

If frequency is to be kept at f , then by governor action reduce the prime mover torque of alternator 2, so that frequency is again become f . In this case load supplied by generators will be P''_1 and P''_2 .

TWO GENERATORS IN PARALLEL – POTTERY WHEEL ANALOGY

ANALOGY



Pottery Wheel
= Power System



Wheel Speed (RPM)
= System Frequency (Hz)



Person A
= Generator 1



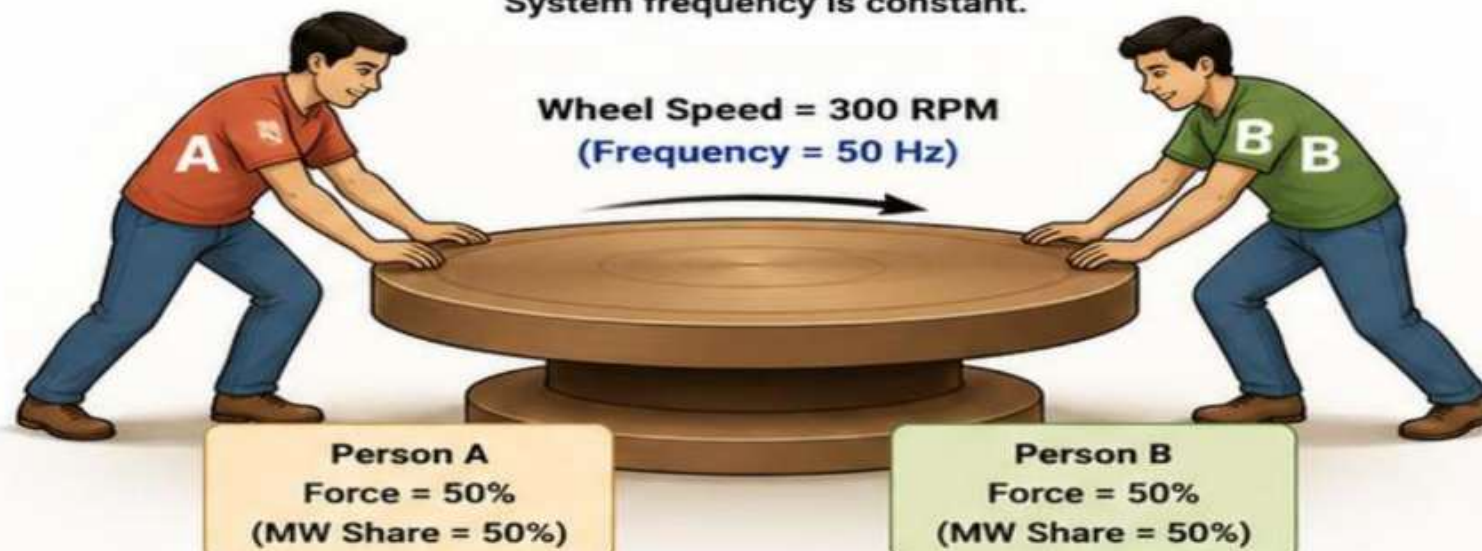
Person B
= Generator 2



Hand Force
= Mechanical Input
(Steam Input)

1) INITIAL CONDITION

Both generators are supplying power.
System frequency is constant.

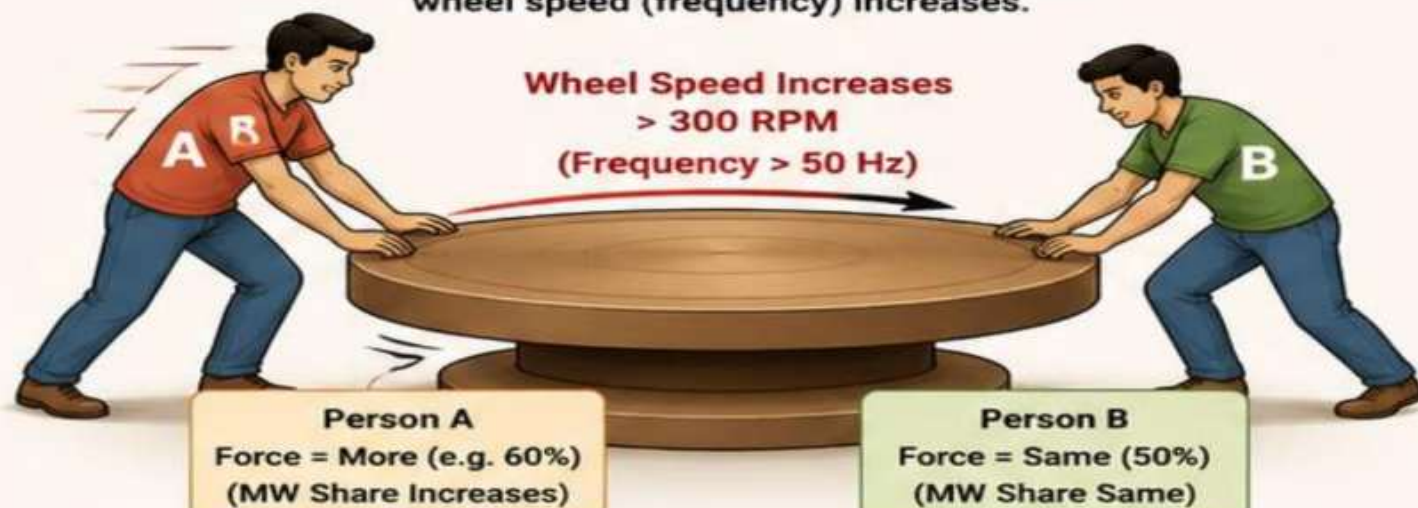


MAPPING TO POWER SYSTEM

Pottery Wheel	Power System
RPM of Wheel	System Frequency (Hz)
Hand Force	Mechanical Input (Steam Input)
Person A	Generator 1
Person B	Generator 2
More Force by A	Increase in Mechanical Input of Gen 1
Less Force by B	Decrease in Mechanical Input of Gen 2
Constant Wheel Speed	Constant System Frequency
Sharing of Force	Load Sharing (MW Sharing)

2) INCREASE MECHANICAL INPUT OF GENERATOR A

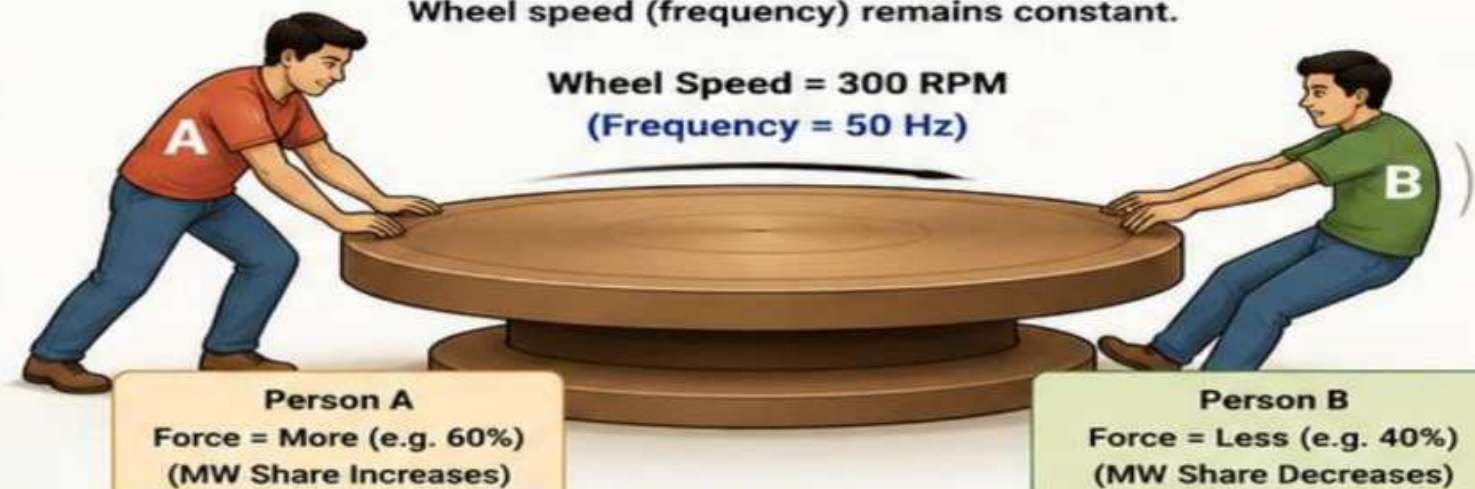
Person A pushes harder.
If Person B continues with same force,
wheel speed (frequency) increases.



Result: Frequency increases (System becomes faster)

3) REDUCE MECHANICAL INPUT OF GENERATOR B TO MAINTAIN SAME FREQUENCY

Person B reduces his force (steam input).
Now total force is same as before.
Wheel speed (frequency) remains constant.

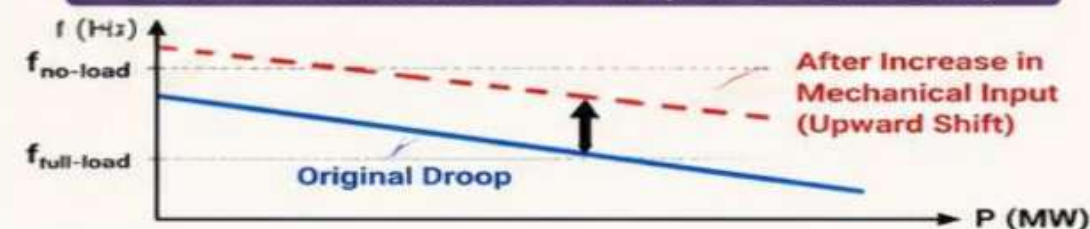


Result: Frequency remains same, but load sharing changes.

KEY TAKEAWAYS

- Increasing mechanical input of one generator shifts its droop characteristic upward.
- If the other generator does not change, system frequency will change.
- To keep system frequency constant, the other generator must reduce its mechanical input.
- This changes the load (MW) sharing between the two generators, but total generation remains the same.

DROOP CHARACTERISTIC SHIFT (FOR GENERATOR A)





2 Minutes Summary

→ ELD with loss

$$(Ic)L_1 = \lambda$$

→ Action of governor.

Thank
you

PRACTICE SHEET-1

EE

POWER SYSTEM

Economic load dispatch

Q1 In a simple two-unit system, the loss coefficients are:

$B_{11} = 0.008$ per MW, $B_{12} = -0.0025$ per MW and $B_{22} = 0.012$ per MW. The incremental production cost of two units are :

$$\frac{dC_1}{dP_1} = 0.075 P_1 + 20$$

$$\text{and } \frac{dC_2}{dP_2} = 0.08 P_2 + 16 \text{ Rs/MWh}$$

The power generations P_1 and P_2 for $\lambda = 25$ is

(A) $P_1 = 15.94$ MW, $P_2 = 14.72$ MW

(B) $P_1 = 14.72$ MW, $P_2 = 15.94$ MW

(C) $P_1 = 12.52$ MW, $P_2 = 14.86$ MW

(D) $P_1 = 14.86$ MW, $P_2 = 12.52$ MW

Q2 A generator is supplying a load. An incremental change in load of 10 MW requires generation to be increased to 15 MW. The cost at plant bus is Rs 40/MWhr. The incremental cost at the receiving end is Rs :

(A) 30/MWhr

(B) 45/MWhr

(C) 60/MWhr

(D) 90/MWhr

Q3 The incremental cost characteristics of two generators delivering 250 MW are as follows :

$$\frac{dF_1}{dP_1} = 2.7 + 0.015P_1, \quad \frac{dF_2}{dP_2} = 1.9 + 0.024P_2$$

For economic operation, the generations P_1 and P_2 should be :

(A) $P_1 = P_2 = 125$ MW

(B) $P_1 = 133.33$ MW, $P_2 = 116.66$ MW

(C) $P_1 = 116.66$ MW, $P_2 = 133.33$ MW

(D) $P_1 = 150$ MW, $P_2 = 100$ MW

Q4 A power system has two generators with the following cost curves:

Generator 1:

$$C_1(P_1) = 0.008P_1^2 + 12P_1 + 300$$

(Thousand Rupees/Hour)

Generator 2:

$$C_2(P_2) = 0.008P_2^2 + 10P_2 + 500$$

(Thousand Rupees/Hour)

The generator limits are

$$120 \text{ MW} \leq P_1 \leq 550 \text{ MW}$$

$$100 \text{ MW} \leq P_2 \leq 600 \text{ MW}$$

A load demand of 525 MW is supplied by the generators in an optimal manner. Neglecting losses in the transmission network, the optimal generation of each generator is :

(A) $P_1 = 200$ MW, $P_2 = 325$ MW

(B) $P_1 = 325$ MW, $P_2 = 200$ MW

(C) $P_1 = 275$ MW, $P_2 = 250$ MW

(D) $P_1 = 250$ MW, $P_2 = 275$ MW

Q5 In a power system, the fuel inputs per hour of plants 1 and 2 are given as



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$$F_1 = 0.25P_1^2 + 25P_1 + 150 \text{ Rs per hour}$$

$$F_2 = 0.2P_2^2 + 55P_2 + 200 \text{ Rs per hour}$$

The limits generators are:

$$50 \leq P_1 \leq 100 \text{ MW}$$

$$60 \leq P_2 \leq 250 \text{ MW}$$

Find the economic operating schedule of generation, if the load demand is 150 MW, neglecting transmission losses.

(A) $P_1 = 100 \text{ MW}, P_2 = 50 \text{ MW}$

(B) $P_1 = 50 \text{ MW}, P_2 = 100 \text{ MW}$

(C) $P_1 = 90 \text{ MW}, P_2 = 60 \text{ MW}$

(D) $P_1 = 60 \text{ MW}, P_2 = 90 \text{ MW}$

- Q6** In a power system, the fuel inputs per hour of plants 1 and 2 are given as

$$F_1 = 0.4P_1^2 + 36P_1 + 250 \text{ Rs per hour}$$

$$F_2 = 0.25P_2^2 + 42P_2 + 250 \text{ Rs per hour}$$

The limits generators are:

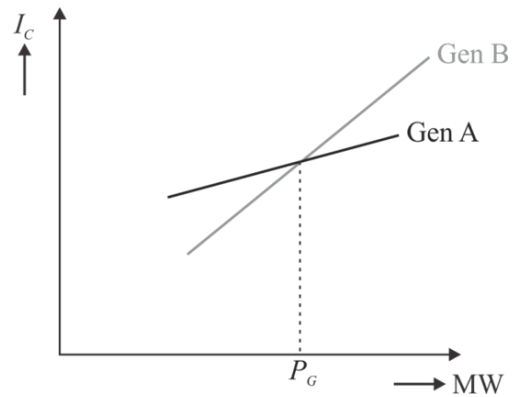
$$80 \leq P_1 \leq 150 \text{ MW and } 50 \leq P_2 \leq 200 \text{ MW}$$

If the load demand is 170 MW, neglecting transmission losses. Then, which of the following is correct for the economic operating schedule of generation?

(A) $P_1 = 70 \text{ MW}$ (B) $P_2 = 100 \text{ MW}$

(C) $P_1 = 80 \text{ MW}$ (D) $P_2 = 120 \text{ MW}$

- Q7** Figure show the incremental fuel cost curve of generator A and generator B. which of the following statements is **NOT** correct ?



- (A) For more than $2P_G$ power, Generator A will share more load than Generator B
 (B) For less than $2P_G$ power, Generator A will share more load than Generator B
 (C) For $2P_G$ load, both generators will share equal load
 (D) For less than $2P_G$ power, Generator B will share more load than generator A

- Q8** The incremental of two generators delivering 240 MW are as follows :

$$\frac{dF_1}{dP_1} = 30 + 0.15P_1, \quad \frac{dF_2}{dP_2} = 20 + 0.25P_2$$

For economic operation, the generation P_1 and P_2 should be.

(A) $P_1 = P_2 = 120 \text{ MW}$

(B) $P_1 = 50 \text{ MW}, P_2 = 190 \text{ MW}$

(C) $P_1 = 125 \text{ MW}, P_2 = 115 \text{ MW}$

(D) $P_1 = 100 \text{ MW}, P_2 = 140 \text{ MW}$

- Q9** Two generating units rated 240 MW and 360 MW have governor speed regulations of 5% and 4.5% respectively from no load to full load. Both the generation units are operating in parallel to share a load of 480 MW. Assuming free governor action, the load shared by the larger units is _____ MW.



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Q10 The combined frequency regulation of machines in an area of capacity 500 MW and operating at

a normal frequency of 50 Hz is 2.5%. The regulation in Hz/MW will be _____.



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Answer Key

Q1 (B)

Q2 (C)

Q3 (B)

Q4 (A)

Q5 (C)

Q6 (C)

Q7 (B)

Q8 (C)

Q9 299~301

Q10 0.002~0.003



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Power System

Economic load dispatch
Practice Sheet 01
Discussion Notes

By- Ajay Gupta Sir

Q.

In a simple two-unit system, the loss coefficients are:

$B_{11} = 0.008$ per MW, $B_{12} = -0.0025$ per MW and $B_{22} = 0.012$ per MW.

The incremental production cost of two units are:

$$\frac{dC_1}{dP_1} = 0.075P_1 + 20 \text{ and } \frac{dC_2}{dP_2} = 0.08P_2 + 16 \text{ Rs/MWh}$$

The power generations P_1 and P_2 for $\lambda = 25$ is

- A** $P_1 = 15.94$ MW, $P_2 = 14.72$ MW
- B** $P_1 = 14.72$ MW, $P_2 = 15.94$ MW
- C** $P_1 = 12.52$ MW, $P_2 = 14.86$ MW
- D** $P_1 = 14.86$ MW, $P_2 = 12.52$ MW

$$P_L = B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2$$

$$\rightarrow P_L = 0.008 P_1^2 + 0.012 P_2^2 - 0.005 P_1 P_2$$

$$\frac{\partial P_L}{\partial P_1} = 0.016 P_1 + 0 - 0.005 P_2$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - (0.016 P_1 - 0.005 P_2)} = \frac{1}{1 - 0.016 P_1 + 0.005 P_2}$$

$$\frac{\partial P_L}{\partial P_2} = 0 + 0.024 P_2 - 0.005 P_1$$

$$L_2 = \frac{1}{1 - \frac{\partial PL}{\partial P_2}} = \frac{1}{1 - (0.024 P_2 - 0.005 P_1)}$$

$$= \frac{1}{1 - 0.024 P_2 + 0.005 P_1}$$

$$\rightarrow (IC)_1 L_1 = \lambda$$

$$(0.075 P_1 + 20) \frac{1}{(1 - 0.016 P_1 + 0.005 P_2)} = 25$$

$$0.075 P_1 + 20 = 25 - 0.4 P_1 + 0.125 P_2$$

$$0.475 P_1 - 0.125 P_2 = 5 \quad - \textcircled{1}$$

$$(IC)_2 L_2 = 1$$

$$(0.08 P_2 + 16) \times \frac{1}{1 - 0.024 P_2 + 0.505 P_1} = 25$$

$$0.08 P_2 + 16 = 25 - 0.6 P_2 + 0.125 P_1$$

$$-0.125 P_1 + 0.68 P_2 = 9 \quad - \textcircled{2}$$

from eqn ① and ②

$$P_1 = 14.72$$

$$P_2 = 15.94$$

Q. A generator is supplying a load. An incremental change in load of 10 MW requires generation to be increased to 15 MW. The cost at plant bus is Rs 40/MWhr. The incremental cost at the receiving end is Rs

- A** 30/MWhr
- B** 45/MWhr
- C** ~~60/MWhr~~
- D** 90/MWhr

$$40 + \frac{50 \times 40}{100} = 60/\text{MWhr}$$



The incremental cost characteristics of two generators delivering 250 MW are as follows



$$\frac{dF_1}{dP_1} = 2.7 + 0.015P_1, \frac{dF_2}{dP_2} = 1.9 + 0.024P_2$$

For economic operation, the generations P_1 and P_2 should be

- A** $P_1 = P_2 = 125$ MW
- B** $P_1 = 133.33$ MW, $P_2 = 116.66$ MW
- C** $P_1 = 116.66$ MW, $P_2 = 133.33$ MW
- D** $P_1 = 150$ MW, $P_2 = 100$ MW

$$P_1 + P_2 = 250 \quad - (1)$$

$$2.7 + 0.015P_1 = 1.9 + 0.024P_2$$

$$0.015P_1 - 0.024P_2 = -0.8$$

$$15P_1 - 24P_2 = -800 \quad - (2)$$

from (1) and (2) $P_1 = 133.33$ $P_2 = 116.66$

Q.

A power system has two generators with the following cost curves:

Generator 1: $C_1(P_1) = 0.008P_1^2 + 12P_1 + 300$ (Thousand Rupees/Hour)

Generator 2: $C_2(P_2) = 0.008P_2^2 + 10P_2 + 500$ (Thousand Rupees/Hour)

The generator limits are:

$$120 \text{ MW} \leq P_1 \leq 550 \text{ MW}$$

$$100 \text{ MW} \leq P_2 \leq 600 \text{ MW}$$

A load demand of 525 MW is supplied by the generators in an optimal manner. Neglecting losses in the transmission network, the optimal generation of each generator is

A $P_1 = 200 \text{ MW}, P_2 = 325 \text{ MW}$

B $P_1 = 325 \text{ MW}, P_2 = 200 \text{ MW}$

C $P_1 = 275 \text{ MW}, P_2 = 250 \text{ MW}$

D $P_1 = 250 \text{ MW}, P_2 = 275 \text{ MW}$

$$(IC)_1 = \frac{dF_1}{dP_1} = 0.016 P_1 + 12$$

$$(IC)_2 = \frac{dF_2}{dP_2} = 0.016 P_2 + 10$$

$$0.016 P_1 + 12 = 0.016 P_2 + 10$$

$$P_2 - P_1 = \frac{12}{0.016} = 750 - \textcircled{1}$$

$$P_1 + P_2 = 525 - \textcircled{2}$$

$$P_1 =$$

$$P_2 =$$

$$P_1 = 250$$

$$P_2 = 325$$



In a power system, the fuel inputs per hour of plants 1 and 2 are given as

$$F_1 = 0.25P_1^2 + 25P_1 + 150 \text{ Rs per hour}$$

$$F_2 = 0.2P_2^2 + 55P_2 + 200 \text{ Rs per hour}$$

The limits generators are:

$$50 \leq P_1 \leq 100 \text{ MW}$$

$$60 \leq P_2 \leq 250 \text{ MW}$$

Find the economic operating schedule of generation, if the load demand is 150 MW, neglecting transmission losses.

A $P_1 = 100 \text{ MW}, P_2 = 50 \text{ MW}$

B $P_1 = 50 \text{ MW}, P_2 = 100 \text{ MW}$

C ✓ $P_1 = 90 \text{ MW}, P_2 = 60 \text{ MW}$

D $P_1 = 60 \text{ MW}, P_2 = 90 \text{ MW}$

$$\frac{dF_1}{dP_1} = 0.5P_1 + 25$$

$$\frac{dF_2}{dP_2} = 0.4P_2 + 55$$

$$(IC)_1 = (IC)_2$$

$$0.5P_1 + 25 = 0.4P_2 + 55$$

$$0.5P_1 - 0.4P_2 = 30$$

$$5P_1 - 4P_2 = 300 \quad - \textcircled{1}$$

$$P_1 + P_2 = 150 \quad - \textcircled{2}$$

$$\begin{array}{r} 5P_1 - 4P_2 = 300 \\ 4P_1 + 4P_2 = 600 \\ \hline 9P_1 = 900 \\ P_1 = 100 \\ P_2 = 50 \end{array}$$

$$\checkmark P_2(\text{cal}) < P_2(\text{min})$$

$$\text{So } P_2 = P_2(\text{min}) = 60$$

$$P_1 = 150 - 60 = 90$$



In a power system, the fuel inputs per hour of plants 1 and 2 are given as

$$F_1 = 0.4P_1^2 + 36P_1 + 250 \quad \text{Rs per hour}$$

$$F_2 = 0.25P_2^2 + 42P_2 + 250 \quad \text{Rs per hour}$$

The limits generators are:

$$80 \leq P_1 \leq 150 \text{ MW and } 50 \leq P_2 \leq 200 \text{ MW}$$

If the load demand is 170 MW, neglecting transmission losses. Then, which of the following is correct for the economic operating schedule of generation?

A $P_1 = 70 \text{ MW}$

B $P_2 = 100 \text{ MW}$

C $P_1 = 80 \text{ MW}$ ✓

D $P_2 = 120 \text{ MW}$

$$\frac{dF_1}{dP_1} = 0.8P_1 + 36$$

$$\frac{dF_2}{dP_2} = 0.5P_2 + 42$$

$$(IC)_1 = (IC)_2$$

$$0.8P_1 + 36 = 0.5P_2 + 42$$

$$0.8P_1 - 0.5P_2 = 6$$

$$8P_1 - 5P_2 = 60 - \textcircled{1}$$

$$P_1 + P_2 = 170$$

$$\begin{array}{r} 8P_1 - 5P_2 = 60 \\ 5P_1 + 5P_2 = 850 \end{array}$$

$$13P_1 = 910$$

$$\underline{P_1 = 70}$$

$$P_1(\text{cal}) < P_1(\text{win})$$

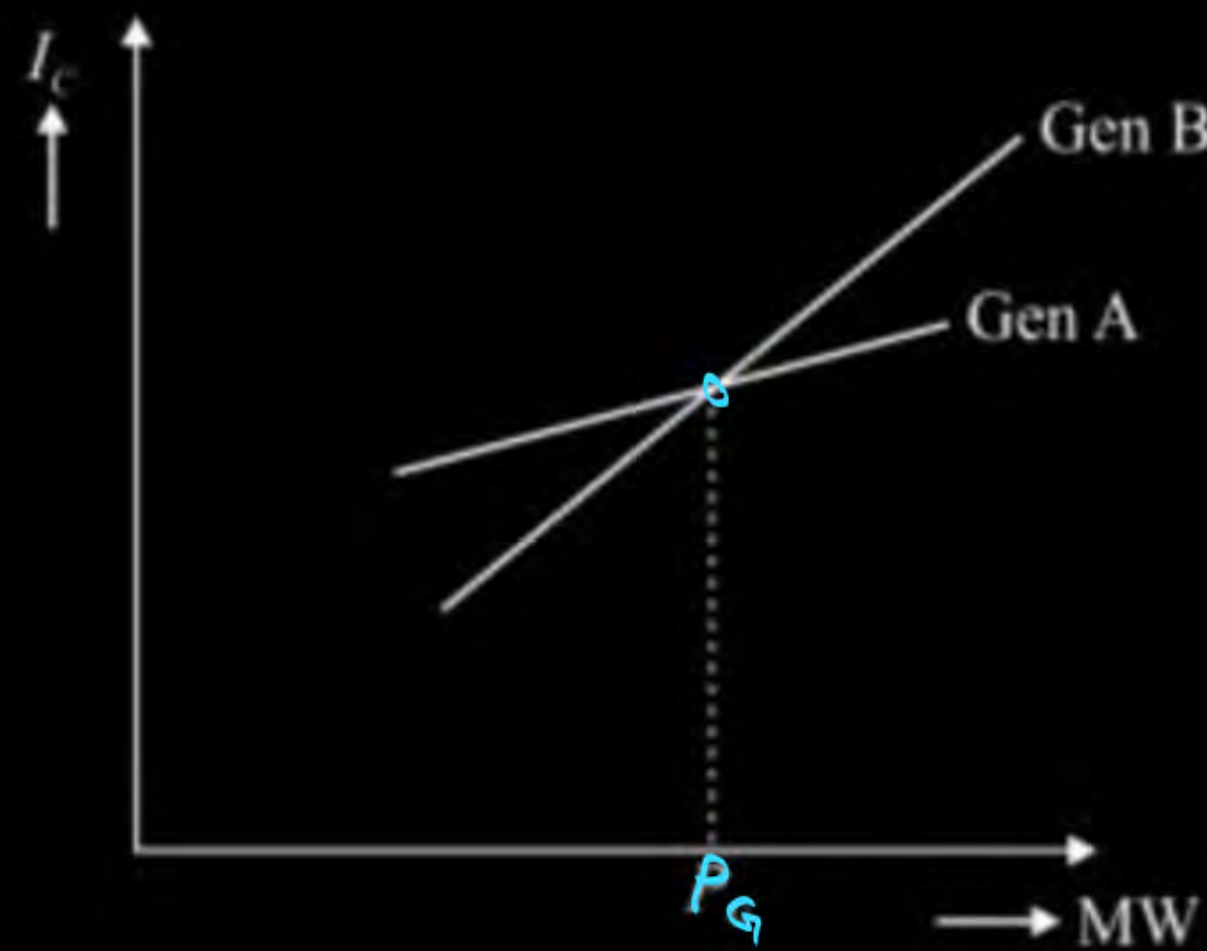
$$\left(\begin{array}{l} P_1 = 80 \\ P_2 = 90 \end{array} \right.$$

Q.



Figure show the incremental fuel cost curve of generator A and generator B. which of the following statements is NOT correct?

- A** For more than $2P_G$ power, Generator A will share more load than Generator B ✓
- B** For less than $2P_G$ power, Generator A will share more load than Generator B ✗
- C** For $2P_G$ load, both generators will share equal load ✓
- D** For less than $2P_G$ power, Generator B will share more load than generator A ✓





The incremental of two generators delivering 240 MW are as follows

$$\frac{dF_1}{dP_1} = 30 + 0.15P_1, \quad \frac{dF_2}{dP_2} = 20 + 0.25P_2$$

For economic operation, the generation P_1 and P_2 should be.

- A** $P_1 = P_2 = 120$ MW
- B** $P_1 = 50$ MW, $P_2 = 190$ MW
- C** $P_1 = 125$ MW, $P_2 = 115$ MW
- D** $P_1 = 100$ MW, $P_2 = 140$ MW

$$P_1 + P_2 = 240 \text{ --- ①} \quad 30 + 0.15P_1 = 20 + 0.25P_2$$

$$0.15P_1 - 0.25P_2 = -10$$

$$15P_1 - 25P_2 = -1000$$

$$3P_1 - 5P_2 = -200 \text{ --- ②}$$

$$5P_1 + 5P_2 = 1200$$

$$3P_1 - 5P_2 = -200$$

$$8P_1 = 1000$$

$$P_1 = 125$$

$$P_2 = 240 - 125 = 115$$

Q.

PW

Two generating units rated 240 MW and 360 MW have governor speed regulations of 5% and 4.5% respectively from no load to full load. Both the generation units are operating in parallel to share a load of 480 MW. Assuming free governor action, the load shared by the larger units is _____ MW.

$$P_1 + P_2 = 480 \quad - (1)$$

$$50 - \frac{0.05}{240} P_1 = 50 - \frac{0.045}{360} P_2$$

$$\frac{0.05 P_1}{\cancel{240} 2} = \frac{0.045 P_2}{\cancel{360} 3}$$

$$15 P_1 - 9 P_2 = 0$$

$$5 P_1 - 3 P_2 = 0 \quad - (2)$$

$$\begin{array}{r} 5P_1 - 3P_2 = 0 \\ 5P_1 + 5P_2 = 480 \times 5 \\ \hline \end{array}$$

$$-8P_2 = -480 \times 5$$

$$P_2 = \frac{60 \times 480}{8} = \underline{\underline{3600}}$$



The combined frequency regulation of machines in an area of capacity 500 MW and operating at a normal frequency of 50 Hz is 2.5%. The regulation in Hz/MW will be _____.



$$2.5\% \text{ of } 50 \text{ Hz} = 0.025 \times 50$$

$$\text{Regulation} = \frac{0.025 \times \cancel{50}}{\cancel{500} 10} = 0.0025 \text{ MW/Hz}$$

Thank
you