

Electrical Engineering

Power System



Lecture No -1

Bus Impedance Matrix



By- Ajay Gupta Sir



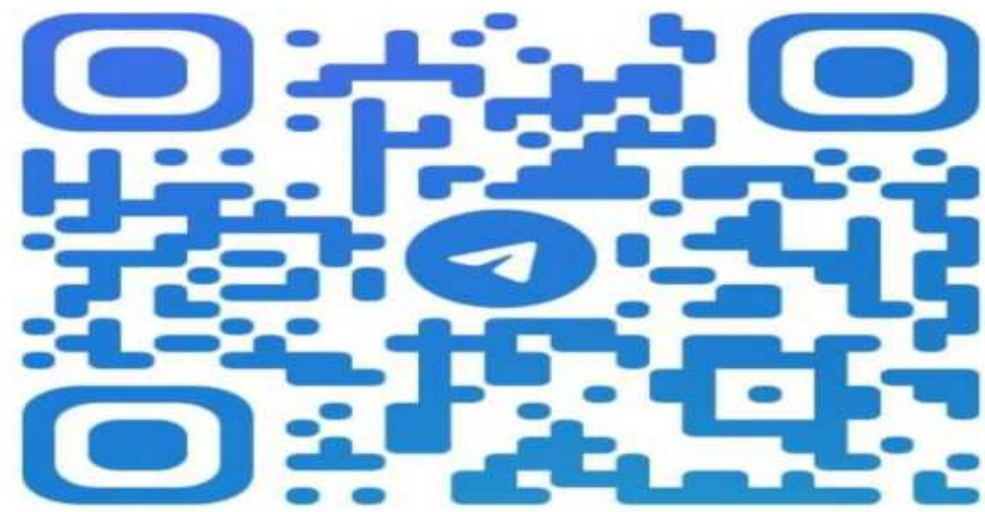
Recap Of previous class

- ✓ 1. Parallel line fault occur near bus
- ✓ 2 one of parallel line is removed
- ✓ 3. Auto reclosing



Topics to be Covered

1. [REDACTED]
2. Method for improvement of T. Stability
- ✓ 3. Impedance matrix
- ✓ 4. Driving point impedance
- ✓ 5. Transfer Impedance



@AJAYGUPTASIRPW

Method to improve transient stability: -

$$\frac{M d^2 \delta}{dt^2} = P_a$$

$$\alpha \downarrow = \frac{d^2 \delta}{dt^2} = \frac{P_a \downarrow}{M \uparrow}$$

$$M = J \left(\frac{2}{P} \right)^2 \omega_s$$

The acceleration should be low, so it allow sufficient time for CB operation.

➤ By Increasing moment of Inertia

$$M \uparrow \propto J \uparrow$$

By designing alternator with large moment of inertia (But it reduce the response of speed governor)

By using fly wheel. It increase moment of Inertia.

➤ By Reducing value of P_a -

$$P_a \downarrow = P_s - P_e \uparrow$$

$$P_a \downarrow = P_s - \left(\frac{EV}{X} \sin \delta \right) \uparrow$$

$$P_e \uparrow = \frac{EV \uparrow}{X \downarrow} \sin \delta$$

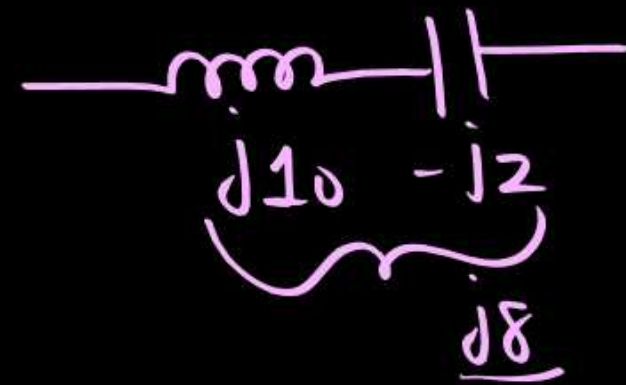
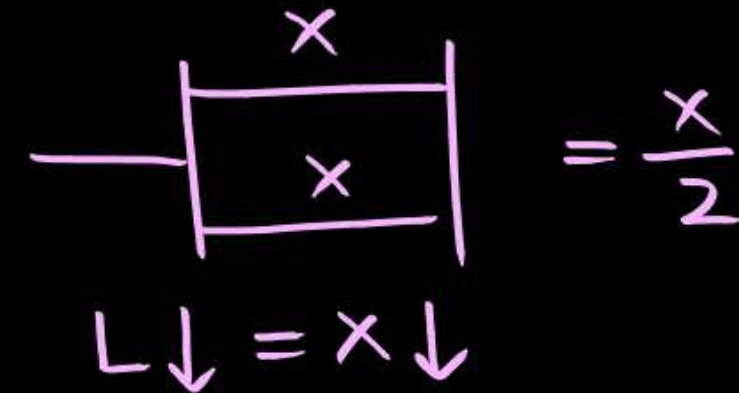
→ By operating the system at high voltage ✓

→ Reduction of Reactance by use of

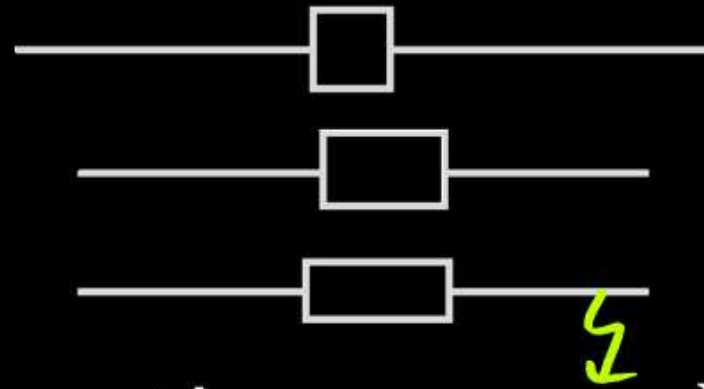
(i) Parallel line (< 320 km)

(ii) Bundle conductor

(iii) Series compensator (> 320 km)

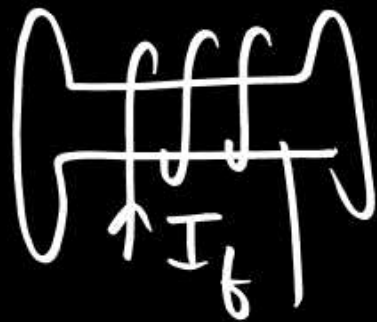


- → Rapid fault clearing by auto enclosure relay.
- High speed CB for rapid fault clearing.
- Quick governor action valve is controlled by electronic valve.
- Single pole switching of CB



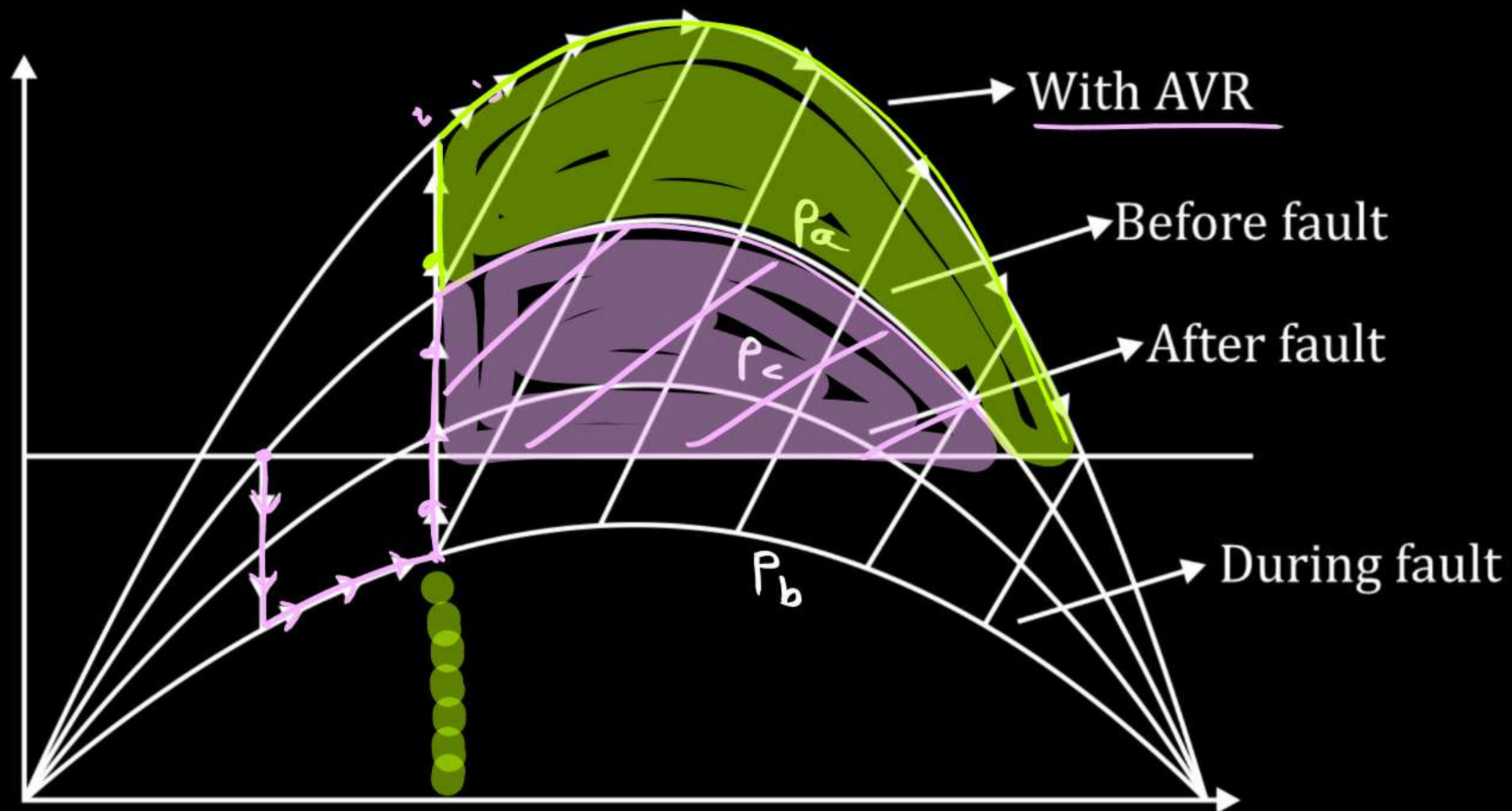
- Fast acting automatic  regulator. (it change field excitation)

Voltage



$$E \propto I_f \uparrow$$

$$P = \frac{EV}{X} \sin \delta$$



Bus Impedance Matrix

$$Y_{12} = Y_{21} \quad Z_{23} = Z_{32}$$

→ Bus impedance matrix Z_{Bus} is mainly used for fault calculation & bus admittance matrix Y_{Bus} is used for load flow analysis.

In very small system Z_{Bus} can be framed from inverse of Y_{Bus} . But in large system this method is rarely used because matrix inverse is become very cumbersome.

→ So Z_{Bus} matrix can be directly constructed, element by element using simple algorithm to incorporate one element at a time.

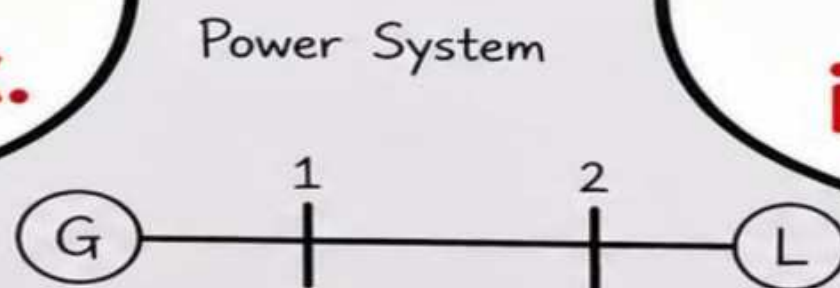
→ Unlike Y_{Bus} , the bus impedance matrix is never sparse.

Both Y_{Bus} & Z_{Bus} is symmetrical around principal diagonal.

$$a_{ij} = a_{ji}$$

In load flow,
we will use the
admittance matrix.

In fault analysis,
we will use the
impedance matrix.



$$Y_{\text{bus}} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Z_{\text{bus}} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Study
Smart
Work
Hard

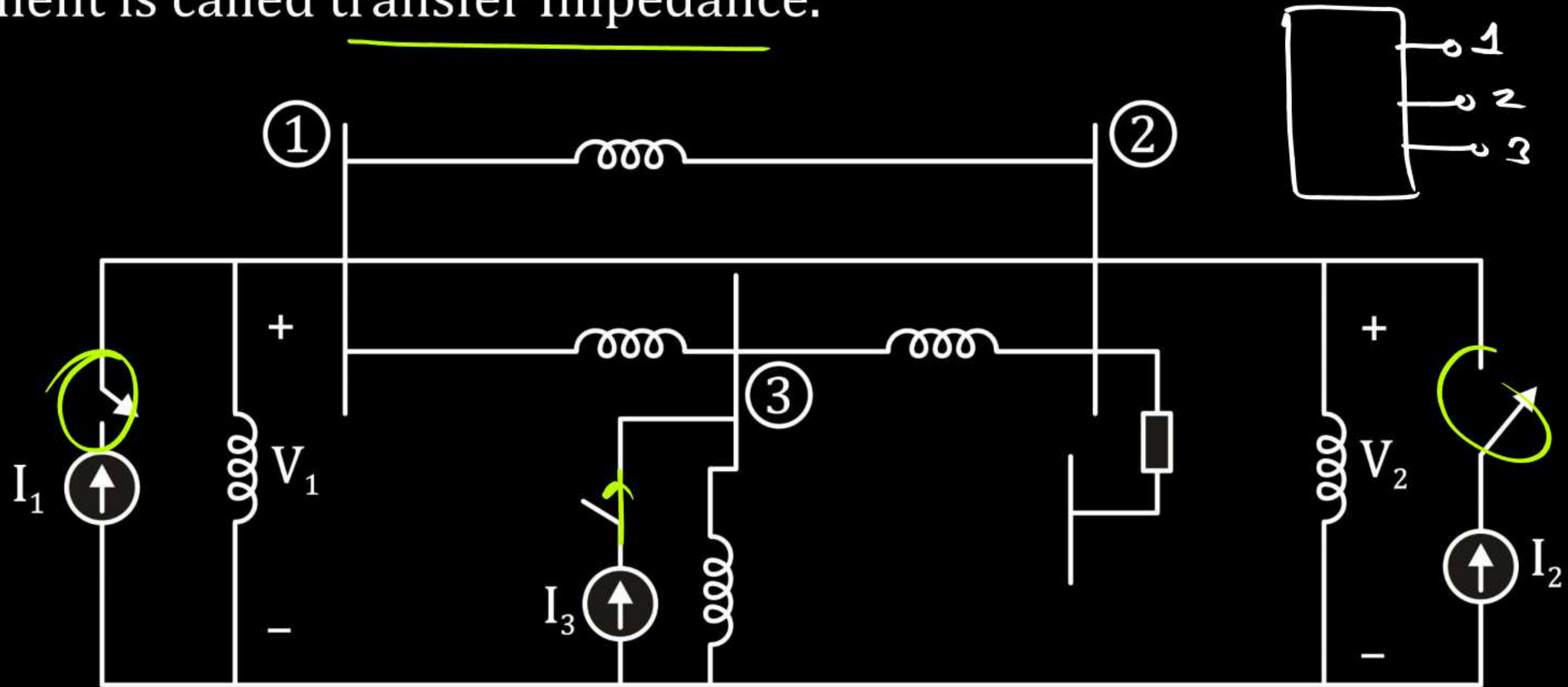
LOAD FLOW

- ✓ Bus Admittance Matrix (Y_{bus})
- ✓ Gauss-Seidel Method
- ✓ Newton-Raphson Method

FAULT ANALYSIS

- ✓ Impedance Matrix (Z_{bus})
- ✓ Short Circuit MVA
- ✓ Fault Current

→ The diagonal element is called driving point impedance & off diagonal element is called transfer Impedance.



$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$V_1 = Z_{11}I_1 + \underline{Z_{12}I_2} + Z_{13}I_3$$

$$\text{if } I_2 = I_3 = 0$$

$$Z_{11} = \frac{V_1}{I_1} \mid I_2 = I_3 = 0$$

$$V_2 = \underline{Z_{21}I_1} + Z_{22}I_2 + \underline{Z_{23}I_3}$$

$$\downarrow$$

$$I_1 = I_3 = 0$$

$$Z_{22} = \frac{V_2}{I_2} \mid I_1 = I_3 = 0$$

$$V_{\text{Bus}} = Z_{\text{Bus}} I_{\text{Bus}}$$

$$Z_{22} = \frac{V_2}{I_2} \bigg/ \begin{matrix} I_1 = 0 \\ I_3 = 0 \end{matrix}$$

So driving point impedance is determined by open circuiting the current source at bus 1 & 3 & injecting I_2 current at bus 2 then ratio of voltage & current at bus 2 will be driving point impedance Z_{22} .

Similarly,

$$V_1 = Z_{12} I_2$$

$$Z_{12} = \frac{V_1}{I_2} \bigg/ \left. \begin{array}{l} I_1 = 0 \\ I_3 = 0 \end{array} \right\}$$

So transfer impedance is the ratio of voltage & current of different buses if all other current are reduce to zero.

→ Always remember,

$$Z_{11} = Z_{1(+n)}$$

$$[Z_{Bus}] = \frac{1}{[Y_{Bus}]^{-1}}$$

But $\underline{Z_{11}} \neq \frac{1}{\underline{Y_{11}}}$

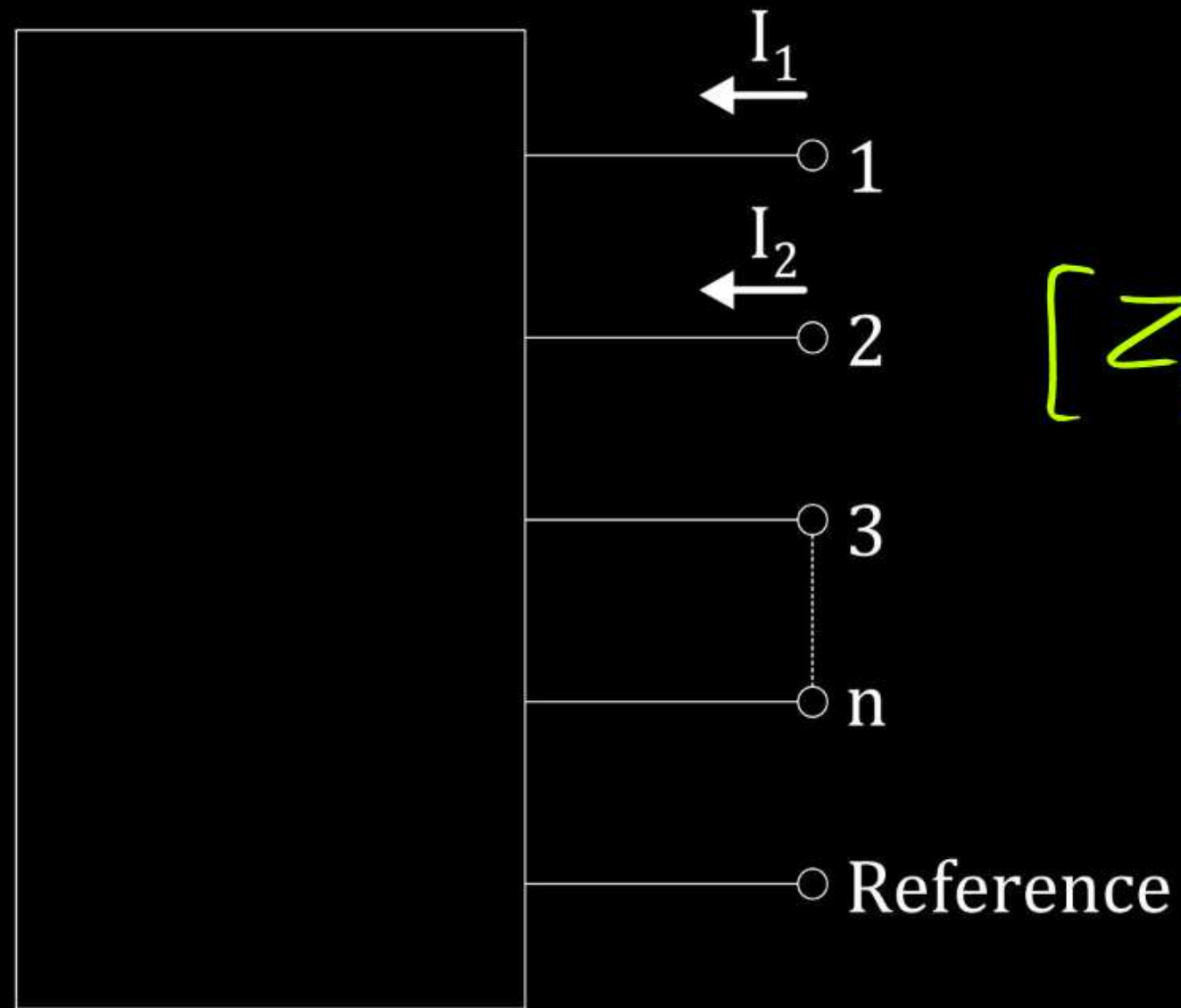
$$Z_{12} \neq \frac{1}{Y_{12}}$$

$$\begin{bmatrix} 3 & 4 \\ 2 & 6 \end{bmatrix}^{-1} \neq \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

→ The diagonal element of Z_{Bus} is the Thevenin equivalent impedance at corresponding Bus.

→ In Z_{Bus} formation always take one bus as a reference bus.

Algorithm of Z_{bus} formation



Total Buses = n
(excluding Ref)

$$[Z_{\text{Bus}}]_{\text{old}} = [n \times n]$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

$$\begin{array}{ccc} [V_{Bus}] = [Z]_{Bus} [I]_{Bus} \\ \downarrow \quad \quad \downarrow \quad \quad \downarrow \\ (n \times 1) \quad (n \times n) \quad (n \times 1) \end{array}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \dots + Z_{1n}I_n$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + \dots + Z_{2n}I_n$$

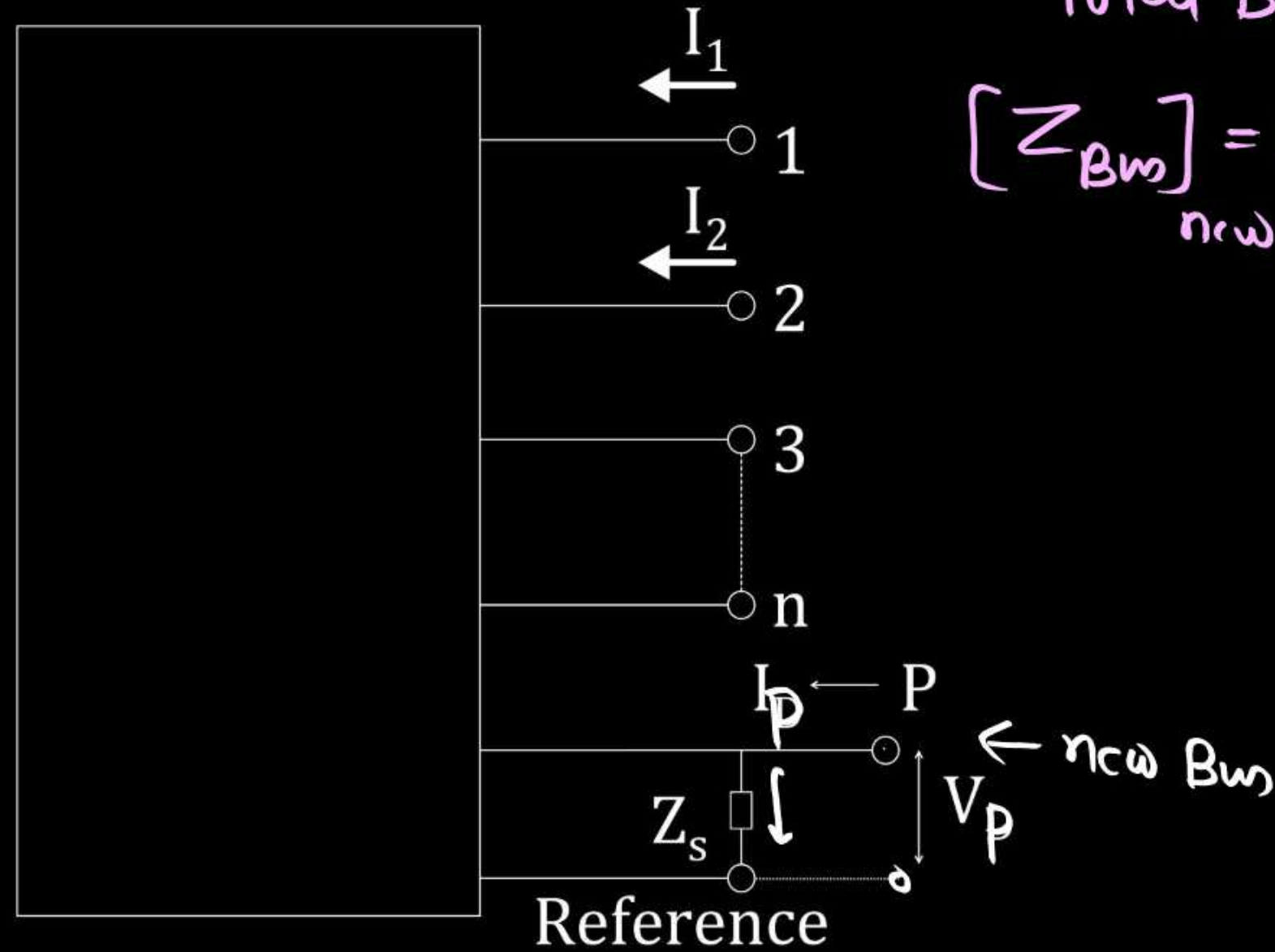
$$\vdots$$

$$V_n = Z_{n1}I_1 + Z_{n2}I_2 + \dots + Z_{nn}I_n$$

Modification of an existing Z_{Bus}

- Suppose original system has n bus so Z_{Bus} has $(n \times n)$ order matrix.
- The existing bus will be represented as $\underbrace{1, 2, 3 \dots l, j, k \dots n}_{\text{existing bus}}$. The letter p and q will be designated to new bus.
- $I_1, I_2 \dots I_n$ current is entering at bus for $1, 2, 3 \dots n$.

Type 1: addition of new element Z_s between new bus p and reference



$$\text{Total Buses} = (n+1)$$

$$[Z_{Bus}]_{new} = (n+1) \times (n+1)$$

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \\ v_p \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} H_1^T \\ H_2^T \\ \vdots \\ H_n^T \\ H_p^T \end{bmatrix}$$

$$\rightarrow V_1 = Z_{11}I_1 + Z_{12}I_2 + \dots + Z_{1n}I_n + 0I_p$$

$$\rightarrow V_2 = Z_{21}I_1 + Z_{22}I_2 + \dots + Z_{2n}I_n + 0I_p$$

⋮

$$\rightarrow V_n = Z_{n1}I_1 + Z_{n2}I_2 + \dots + Z_{nn}I_n + 0I_p$$

$$\rightarrow V_p = I_p Z_s$$

$$V_p = 0I_1 + 0I_2 + \dots + 0I_n + Z_s I_p$$

Now new matrix will be $(n + 1) \times (n + 1)$ order matrix.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ V_p \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & 0 \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & 0 \\ \vdots & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & 0 \\ 0 & 0 & \cdots & 0 & Z_s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_p \end{bmatrix}$$

$\nearrow [Z_{BWO}]_{old}$

$$[Z_{\text{Bus}}]_{\text{new}} = \left[\begin{array}{c|c} [Z_{\text{Bus}}]_{\text{old}} & \begin{matrix} 0 \\ 0 \\ \vdots \end{matrix} \\ \hline \begin{matrix} 0 & 0 & \cdots \end{matrix} & Z_s \end{array} \right]$$

Q. Formed Z_{Bus} Matrix

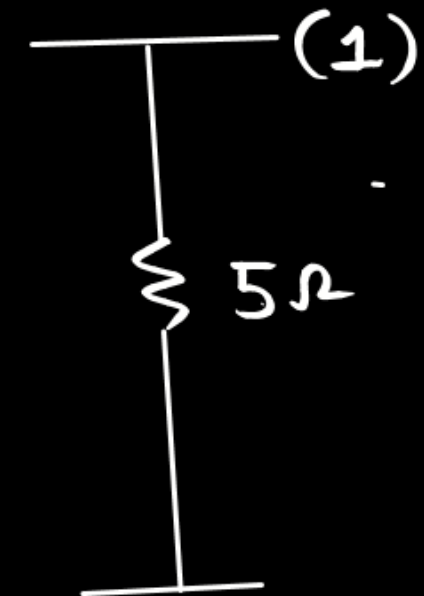
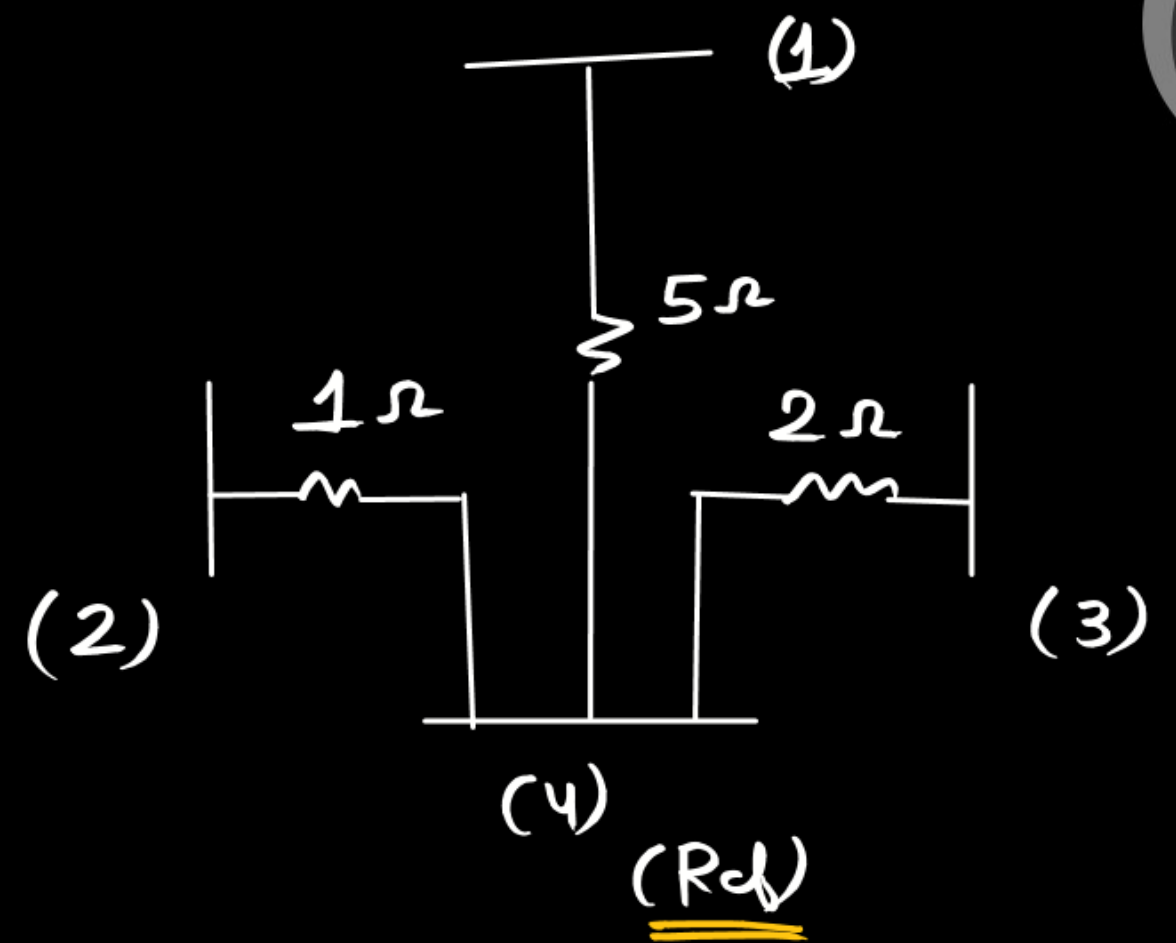
Step-1

Initial No power system exist

$$Z_{Bus} = \text{NIL}$$

Step-2

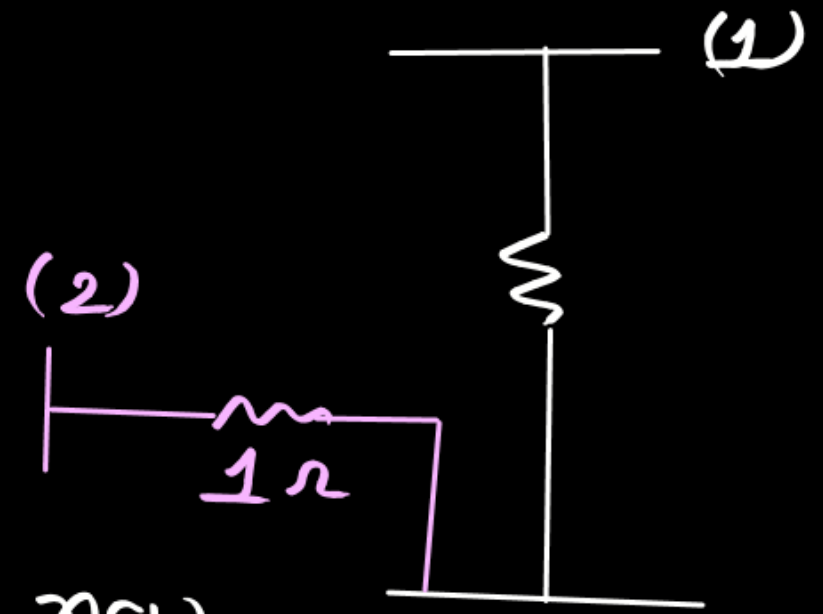
An element $z_s = 5\Omega$ is added Between new Bus(1) and Ref.



$$Z_{Bus} = [5]$$

Step 3

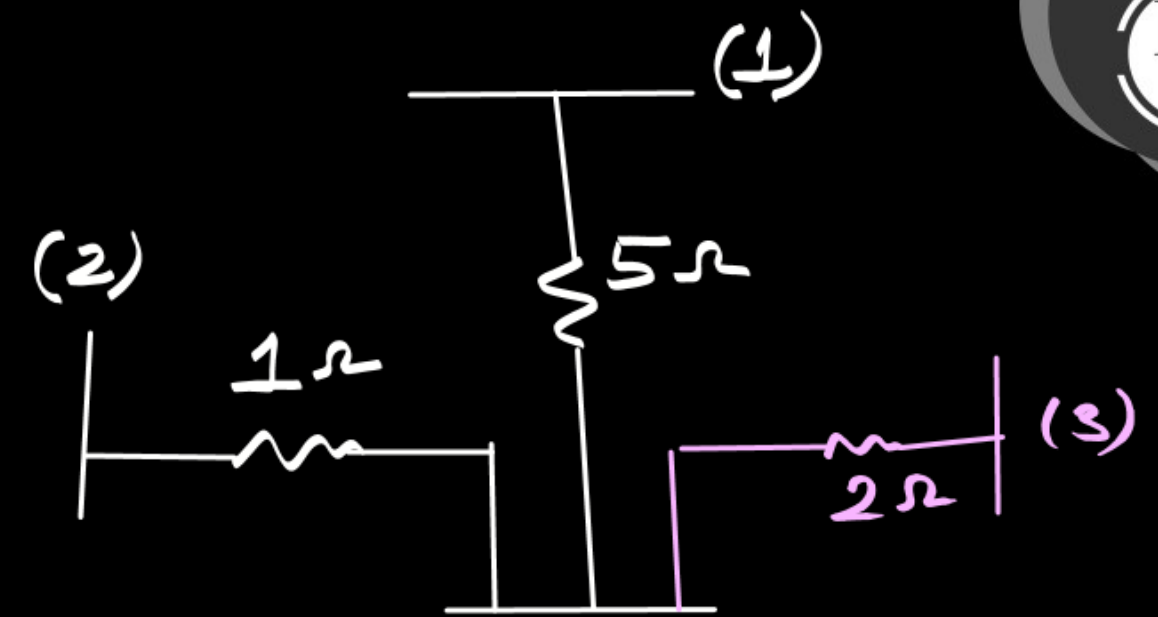
An element $Z_g = 1\Omega$ is added between new Bus 2 and Ref. Bus.



$$\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

Step-4

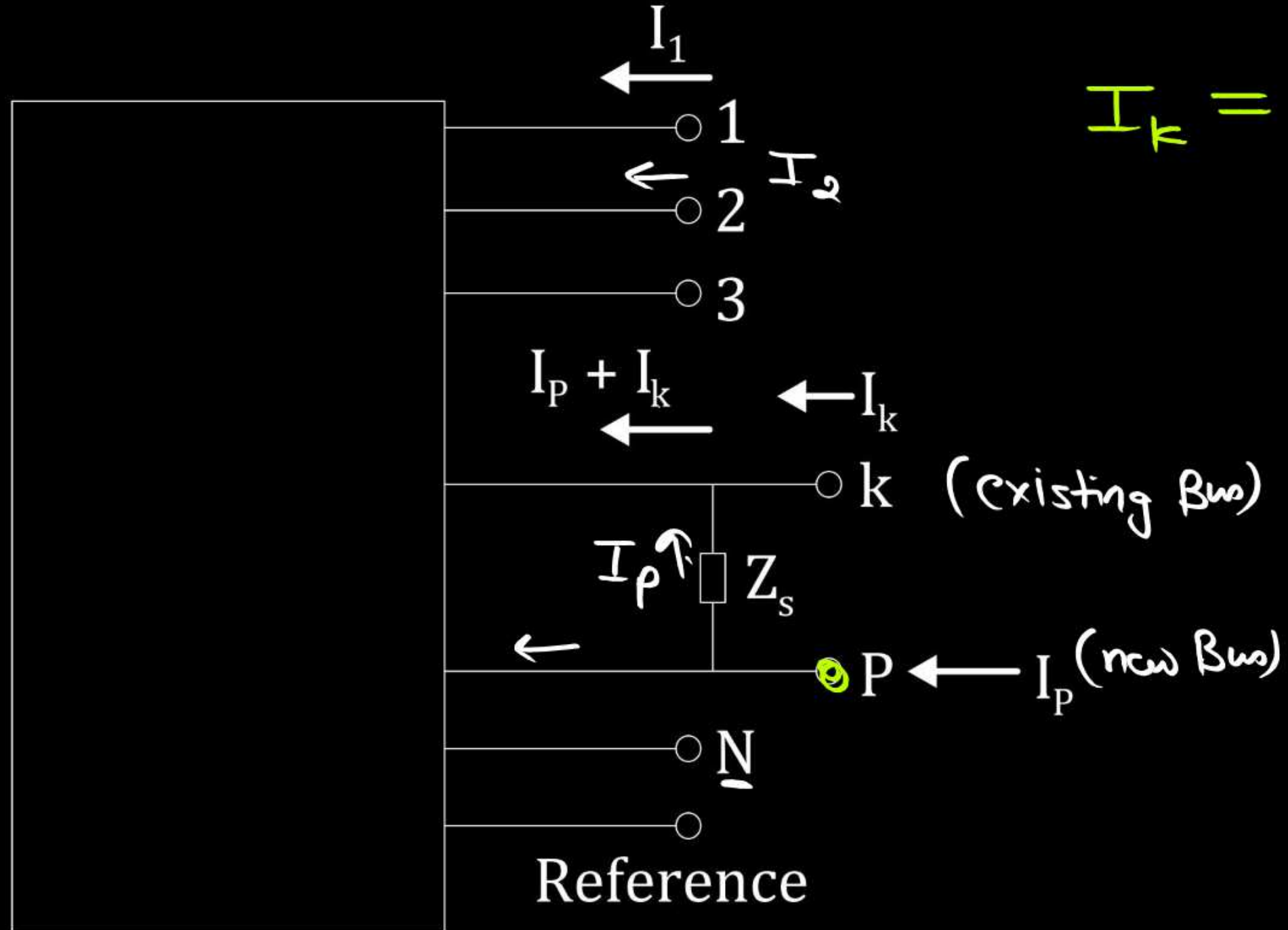
A new element $Z_s = 2\Omega$ is added between new Bus 3 and Ref Bus.



$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Type 2: Addition of new element Z_s between New bus P and existing bus k

$$V_k + I_p \cdot Z_s = V_p$$



$$V_1 = Z_{11}I_1 + Z_{12}I_2 \dots + Z_{1k}(I_k + I_p) \dots + Z_{1n}I_n$$

$$\rightarrow V_1 = \underline{Z_{11}}I_1 + \underline{Z_{12}}I_2 \dots + \underline{Z_{1k}}I_k \dots + \underline{Z_{1n}}I_n + \underline{Z_{1k}}I_p$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \dots + Z_{2k}(I_k + I_p) \dots + Z_{2n}I_n$$

$$\rightarrow V_2 = Z_{21}I_1 + Z_{22}I_2 \dots + Z_{2k}I_k \dots + Z_{2n}I_n + Z_{2k}I_p$$

$$\rightarrow V_3 = Z_{31}I_1 + Z_{32}I_2 + \dots + Z_{3k}I_k \dots + Z_{3n}I_n + Z_{3k}I_p$$

⋮

$$\rightarrow V_k = Z_{k1}I_1 + Z_{k2}I_2 \dots + Z_{kk}I_k \dots + Z_{kn}I_n + Z_{kk}I_p$$

$$V_p = V_k + \underline{I_p Z_s}$$

$$V_p = Z_{k1}I_1 + \dots + Z_{kk}I_k + \dots + Z_{kn}I_n + Z_{kp}I_p + \underline{I_p Z_s}$$

$$\rightarrow V_p = Z_{k1}I_1 + Z_{k2}I_2 + \dots + Z_{kk}I_k + \dots + Z_{kn}I_n + (Z_{kp} + Z_s)I_p$$

→ The new matrix will be $(n + 1) \times (n + 1)$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_n \\ V_p \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1} & Z_{k2} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nk} & \cdots & Z_{nn} \\ \hline Z_{k1} & Z_{k2} & \cdots & Z_{kk} & \cdots & Z_{kn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_k \\ \vdots \\ I_n \\ I_p \end{bmatrix}$$

$\downarrow$ column element
 $\downarrow$ row element
 $\downarrow$ $[Z_{kko}]_{old}$
 $\downarrow$ $Z_{kk} + Z_s$

$$[Z_{\text{Bus}}]_{\text{new}} = \left[\begin{array}{c|c} [Z_{\text{Bus}}]_{\text{old}} & \begin{matrix} Z_{1k} \\ Z_{2k} \\ \vdots \\ Z_{nk} \end{matrix} \\ \hline \begin{matrix} Z_{k1} & Z_{k2} & \cdots & Z_{kn} \end{matrix} & \underline{Z_{kk} + Z_s} \end{array} \right]$$

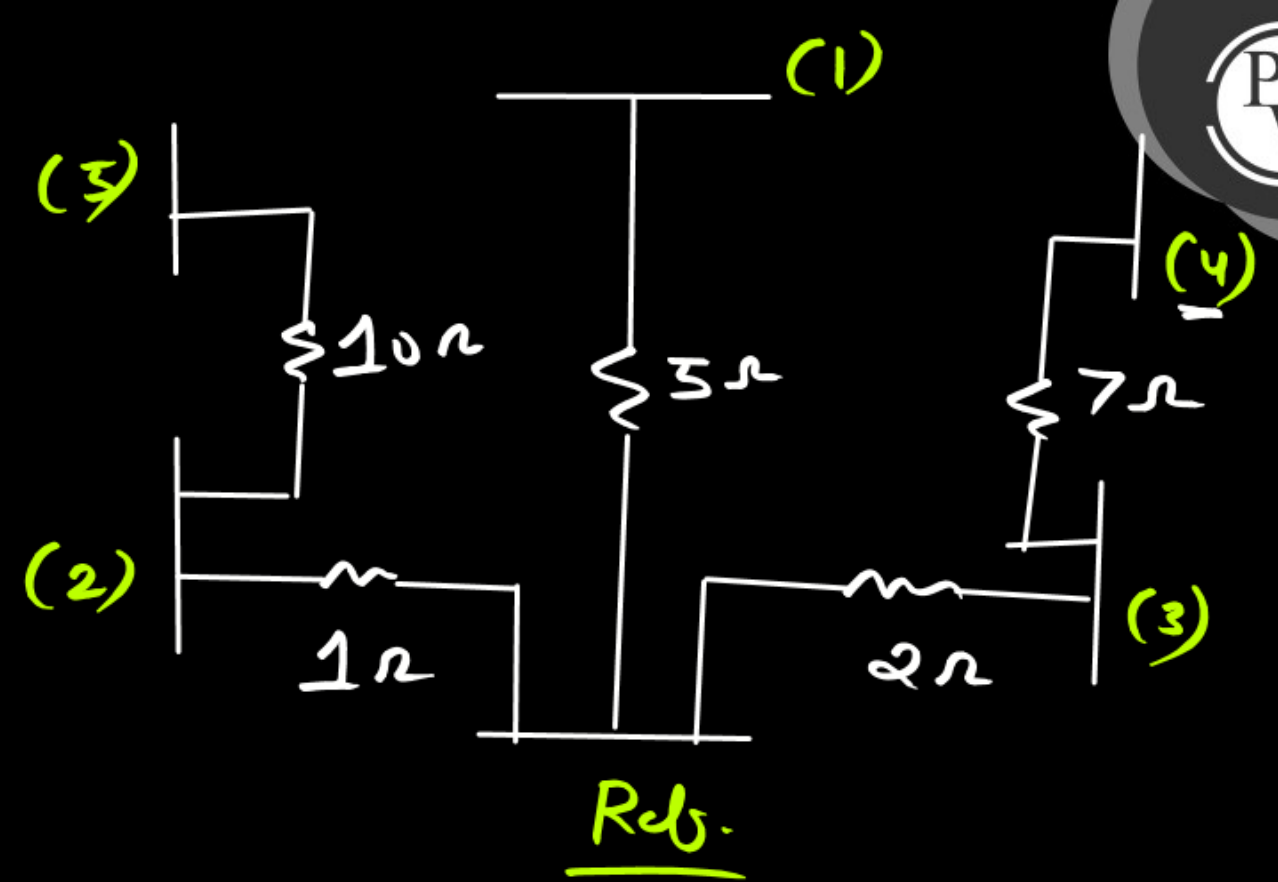
$$= \left[\begin{array}{c|c} [Z_{\text{Bus}}]_{\text{old}} & \leftarrow \text{Column k} \\ \hline \rightarrow \text{Row k} & \underline{Z_{kk} + Z_s} \end{array} \right]$$

Q Find out Z_{Bus} matrix

Step 1

From Q.1

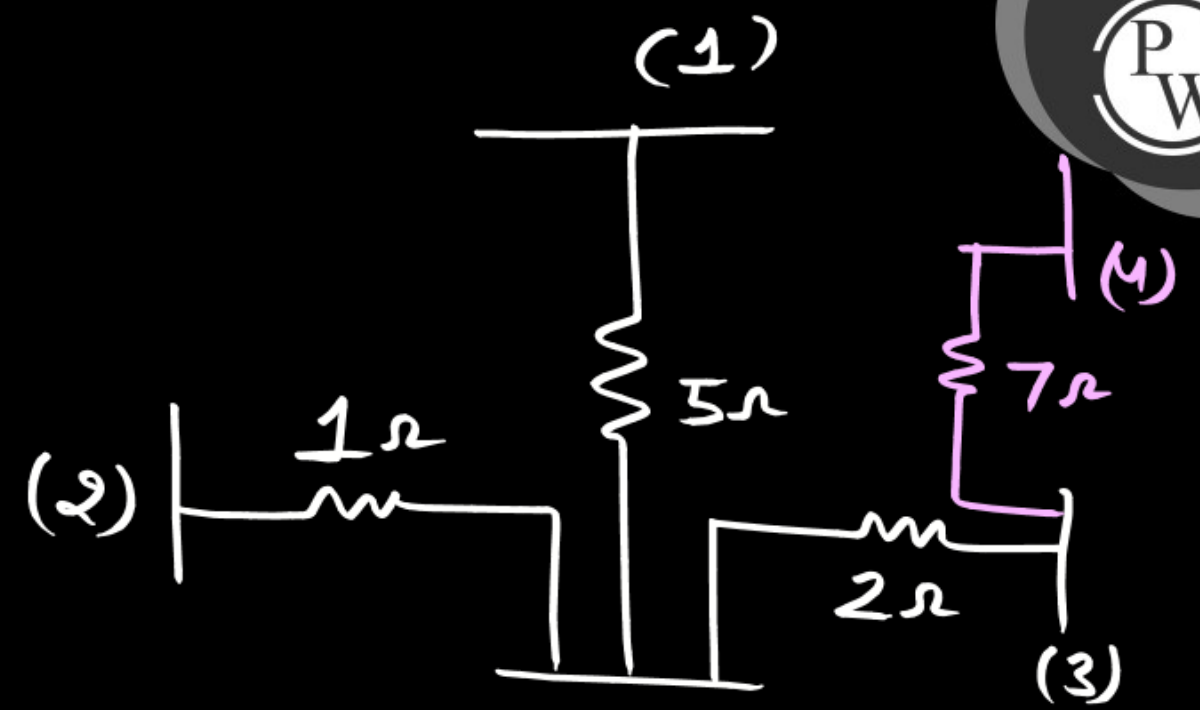
$$Z_{Bus} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$



Step-2

An element $Z_s = 7\Omega$ Between old Bus
 $K=3$ and new Bus $P=4$.

5	0	0	0
0	1	0	0
0	0	2	2
0	0	2	$2+7$ $=9$



$$= Z_{kk} + Z_s$$

$$= Z_{33} + Z_s$$

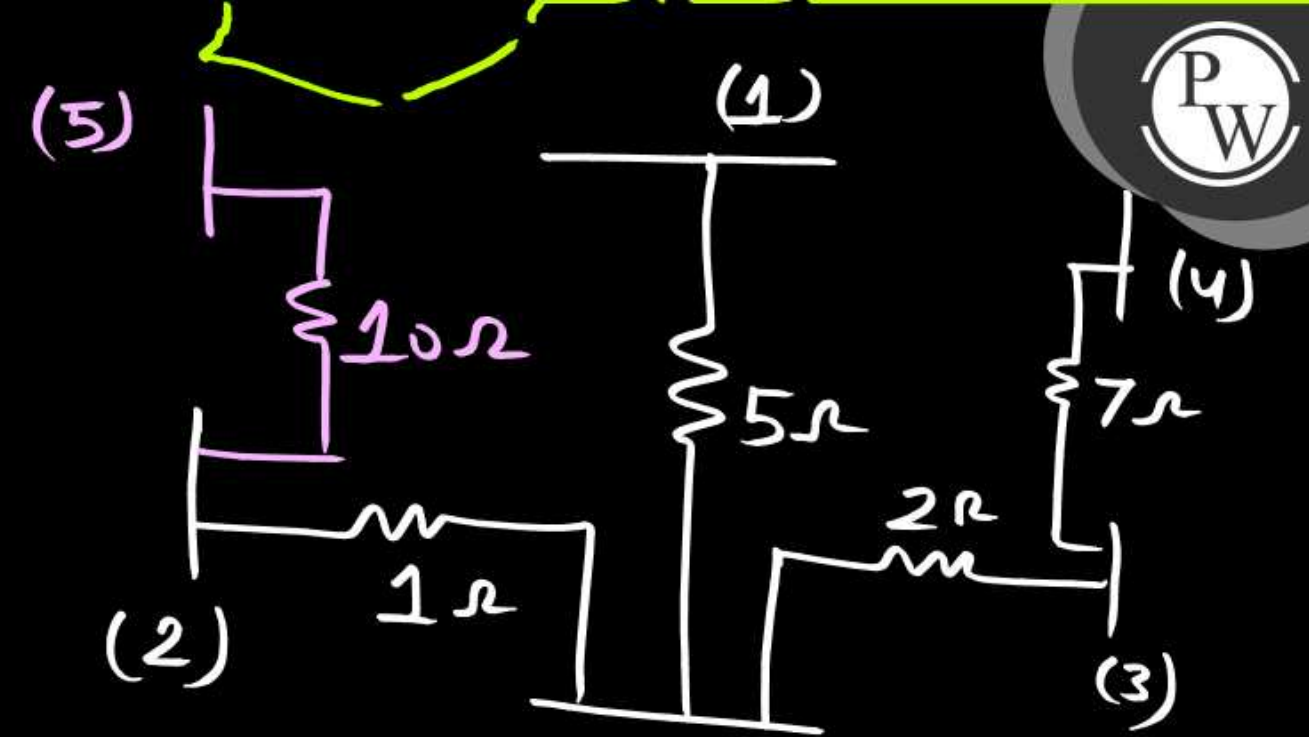
$$= 2 + 7$$

=

Step 3

An element $Z_s = 10\Omega$ is added between old Bus $K=2$ and new Bus $P=5$

5	0	0	0	0
0	1	0	0	1
0	0	2	2	0
0	0	2	9	0
0	1	0	0	$1+10=11$



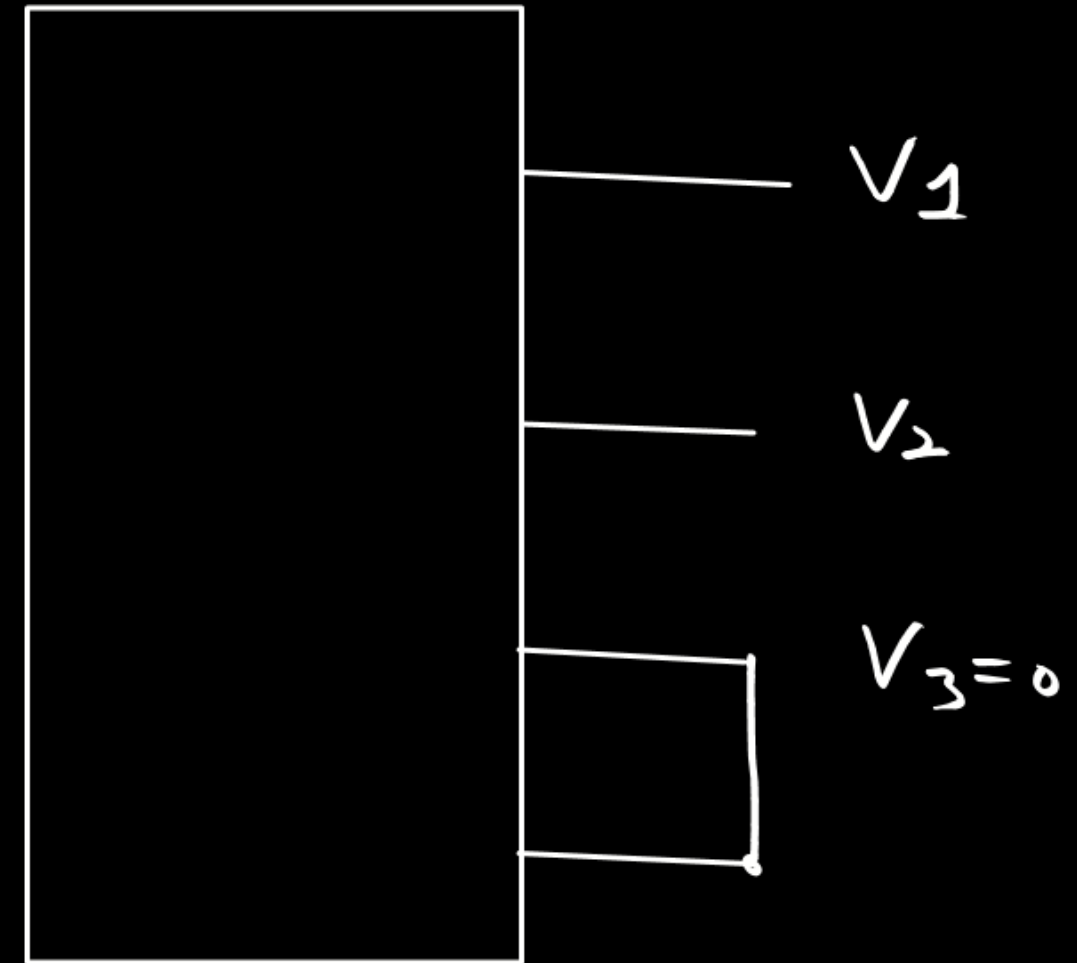
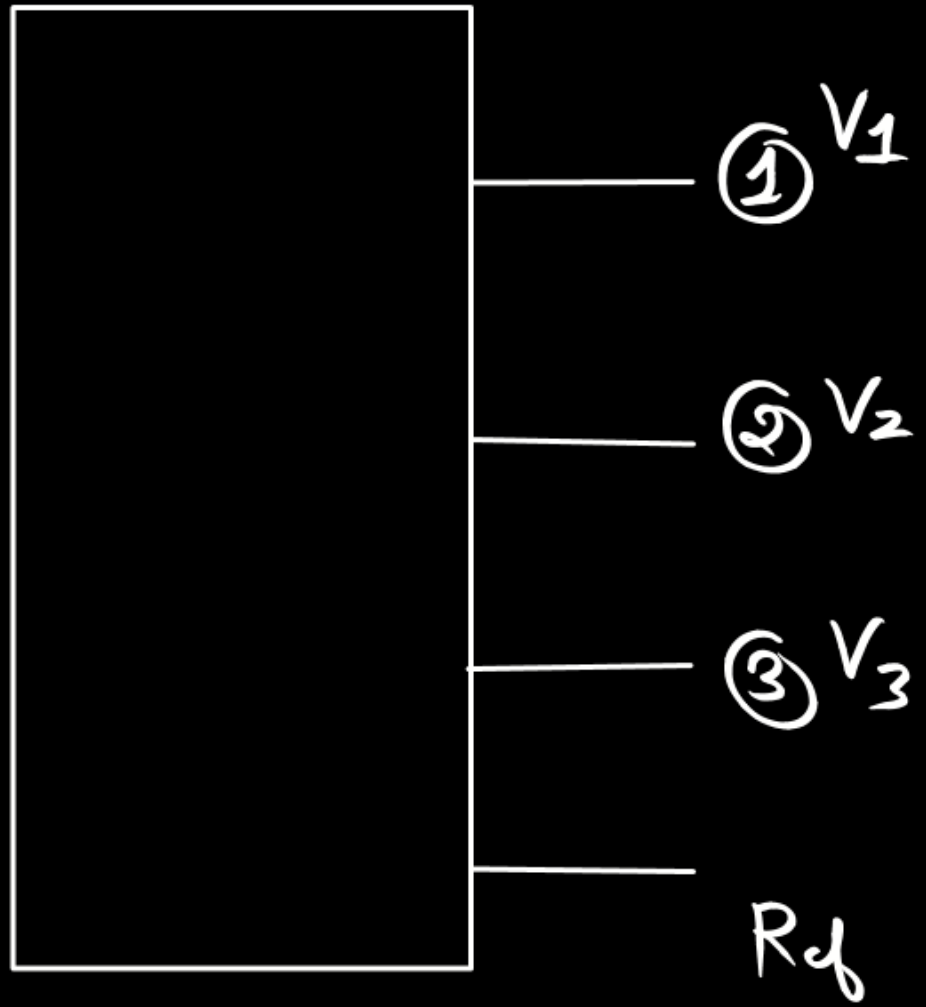
$$Z_{kk} + Z_s$$

$$Z_{22} + Z_s$$

Node elimination in matrix (Kron reduction technique)

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

We want to eliminate ^{one} bus (Node), so that order of Z matrix will reduce $(3 \times 3) \rightarrow (2 \times 2)$



$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \overset{\downarrow}{Z_{13}}I_3 \quad -(1)$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 \quad -(2)$$

$$V_3 = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3$$

Put $V_3 = 0$

$$0 = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3$$

$$I_3 = \frac{-Z_{31}I_1 - Z_{32}I_2}{Z_{33}}$$

Put value of I_3 in eqⁿ (1)

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + Z_{13}\left(\frac{-Z_{31}I_1 - Z_{32}I_2}{Z_{33}}\right)$$

$$V_1 = \left(Z_{11} - \frac{Z_{13}Z_{31}}{Z_{33}}\right)I_1 + \left(Z_{12} - \frac{Z_{13}Z_{32}}{Z_{33}}\right)I_2 \quad - (4)$$

Put value of I_3 in eqⁿ (2)

$$V_2 = Z_{21}I_1 + Z_{22}I_2 + Z_{23}\left(\frac{-Z_{31}I_1 - Z_{32}I_2}{Z_{33}}\right)$$

$$V_2 = \left(Z_{21} - \frac{Z_{23}Z_{31}}{Z_{33}}\right)I_1 + \left(Z_{22} - \frac{Z_{23}Z_{32}}{Z_{33}}\right)I_2 \quad - (5)$$

From eqn (4) and (5)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} - \frac{Z_{13}Z_{31}}{Z_{33}} & Z_{12} - \frac{Z_{13}Z_{32}}{Z_{33}} \\ Z_{21} - \frac{Z_{23}Z_{31}}{Z_{33}} & Z_{22} - \frac{Z_{23}Z_{32}}{Z_{33}} \end{bmatrix}$$

Z

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} - \begin{bmatrix} \frac{Z_{13}Z_{31}}{Z_{33}} & \frac{Z_{13}Z_{32}}{Z_{33}} \\ \frac{Z_{23}Z_{31}}{Z_{33}} & \frac{Z_{23}Z_{32}}{Z_{33}} \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} - \frac{1}{Z_{33}} \begin{bmatrix} \overset{\downarrow 3^{rd} \text{ Col}}{Z_{13}} \\ Z_{23} \end{bmatrix} \begin{bmatrix} Z_{31} & Z_{32} \end{bmatrix}$$

$\nearrow 3^{rd} \text{ Row}$

Suppose in general, if it is required to eliminate $(n + 1)$ row & $(n + 1)$ column.
The new matrix will be,

$$[Z_{new}] = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} - \frac{1}{Z_{(n+1)(n+1)}} \begin{bmatrix} Z_{1(n+1)} \\ Z_{2(n+1)} \\ Z_{n(n+1)} \end{bmatrix} \begin{bmatrix} Z_{(n+1)1} & Z_{(n+1)2} & Z_{(n+1)n} \end{bmatrix}$$

$$\begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} - \frac{1}{z_{33}} \begin{bmatrix} z_{13} \\ z_{23} \end{bmatrix} \begin{bmatrix} z_{31} & z_{32} \end{bmatrix}$$



2 Minutes Summary

- Impedance matrix
- Driving Point & Transfer Impedance
- Modification

Thank
you

Electrical Engineering

Power System



Lecture No 2

Bus Impedance Matrix



By- Ajay Gupta Sir



Recap of previous class

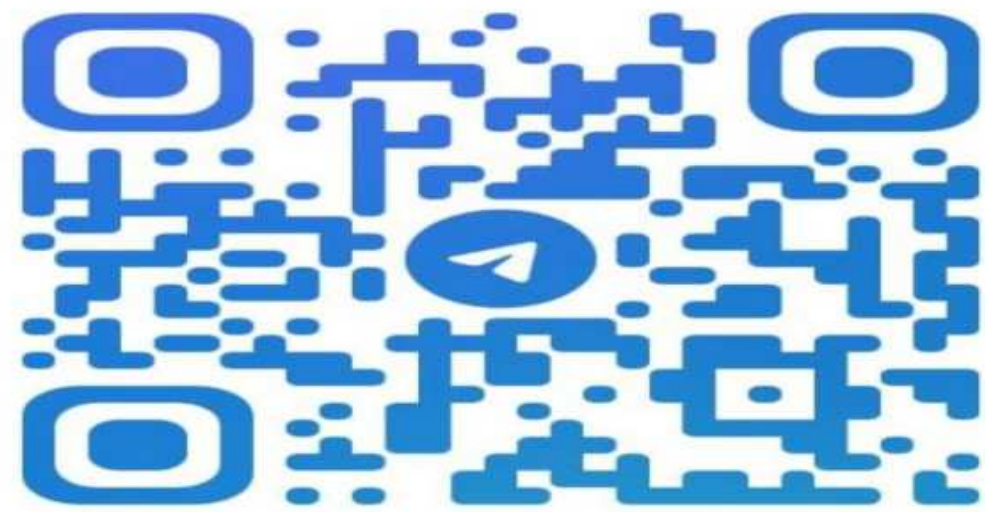


1. Modification of Impedance matrix



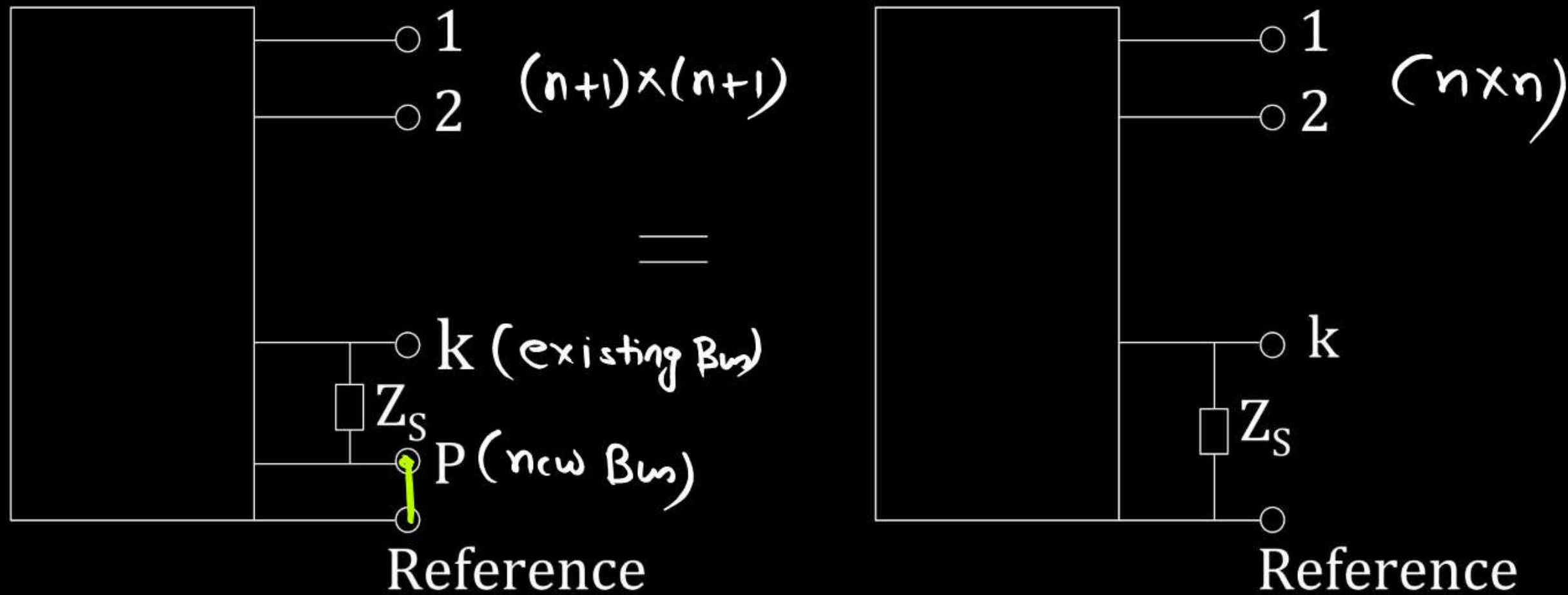
Topics to be Covered

- ✓ 1. Question on Impedance Matrix
- ✓ 2 Fault analysis with impedance matrix



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Type 3: Addition of new element between old bus k and reference



This is the extension of type 2 modification. Initially Z_s is connected between old bus k and new bus P and let new bus P is connected to reference $V_P = 0$

z_{11}	z_{12}	$\dots$	z_{1k}	$\dots$	z_{1n}	z_{1k}
z_{21}	z_{22}	$\dots$	z_{2k}	$\dots$	z_{2n}	z_{2k}
$\vdots$						$\vdots$
z_{k1}	z_{k2}	$\dots$	z_{kk}	$\dots$	z_{kn}	z_{kk}
$\vdots$						$\vdots$
z_{n1}	z_{n2}	$\dots$	z_{nk}	$\dots$	z_{nn}	z_{nk}
z_{k1}	z_{k2}	$\dots$	z_{kk}	$\dots$	z_{kn}	$z_{kk} + z_s$

Adding element z_s
between existing Bus k
and new Bus P

Now Make $V_p = 0$

$$\begin{bmatrix} z_{11} & z_{12} & \dots & z_{1n} \\ z_{21} & z_{22} & \dots & z_{2n} \\ \vdots & & & \\ z_{n1} & z_{n2} & \dots & z_{nn} \end{bmatrix}$$

$$-\frac{1}{(z_{kk} + z_s)}$$

$$\begin{bmatrix} z_{1k} \\ z_{2k} \\ \vdots \\ z_{nk} \end{bmatrix}$$

$$\begin{bmatrix} z_{k1} & z_{k2} & \dots & z_{kn} \end{bmatrix}$$

$$[Z_{Bus}]_{new} = \underline{[Z_{Bus}]_{old}} - \frac{1}{Z_{kk} + Z_s} \begin{bmatrix} Z_{1k} \\ Z_{2k} \\ \vdots \\ Z_{nk} \end{bmatrix} \begin{bmatrix} Z_{k1} & Z_{k2} & \cdots & Z_{kn} \end{bmatrix}$$

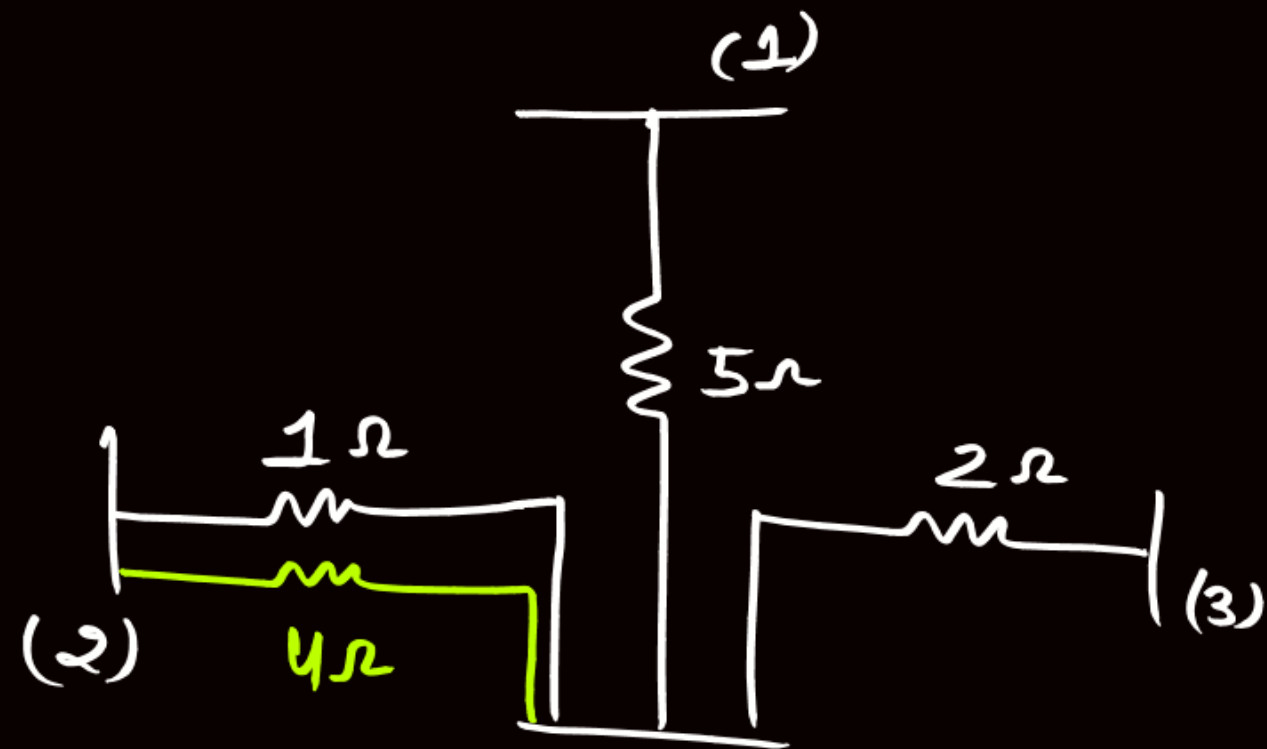



K column

K Row element

Q

$$[Z_{Bus}]_{old} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$



Step - Add an element $Z_s = 4\Omega$ Between existing Bus $k=2$ and Ref Bus.

$$\begin{array}{c} \text{2 Column} \\ \swarrow \end{array}
 \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}
 - \frac{1}{5}$$

2 Rows

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$Z_{kk} + Z_s = Z_{22} + Z_s = 1 + 4$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4/5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Q.

The bus impedance matrix of a 4-bus power system is given by

$$Z_{bus} = \begin{bmatrix} j0.3435 & j0.2860 & j0.2723 & j0.2277 \\ j0.2860 & j0.3408 & j0.2586 & j0.2414 \\ j0.2723 & j0.2586 & j0.2791 & j0.2209 \\ j0.2277 & j0.2414 & j0.2209 & j0.2791 \end{bmatrix}$$

A branch having an impedance of $j0.2\Omega$ is connected between bus 2 and the reference. Then the values of $Z_{22, \text{new}}$ and $Z_{23, \text{new}}$ of the bus impedance matrix of the modified network are respectively

- (a) $j0.5408 \Omega$ and $j0.4586 \Omega$
- ✓ (b) $j0.1260 \Omega$ and $j0.0956 \Omega$
- (c) $j0.5408 \Omega$ and $j0.0956 \Omega$
- (d) $j0.1260 \Omega$ and $j0.1630 \Omega$

$$\begin{aligned} Z_{kk} + Z_s &= Z_{22} + Z_s \\ &= j0.3408 + j0.2 = j0.5408 \end{aligned}$$

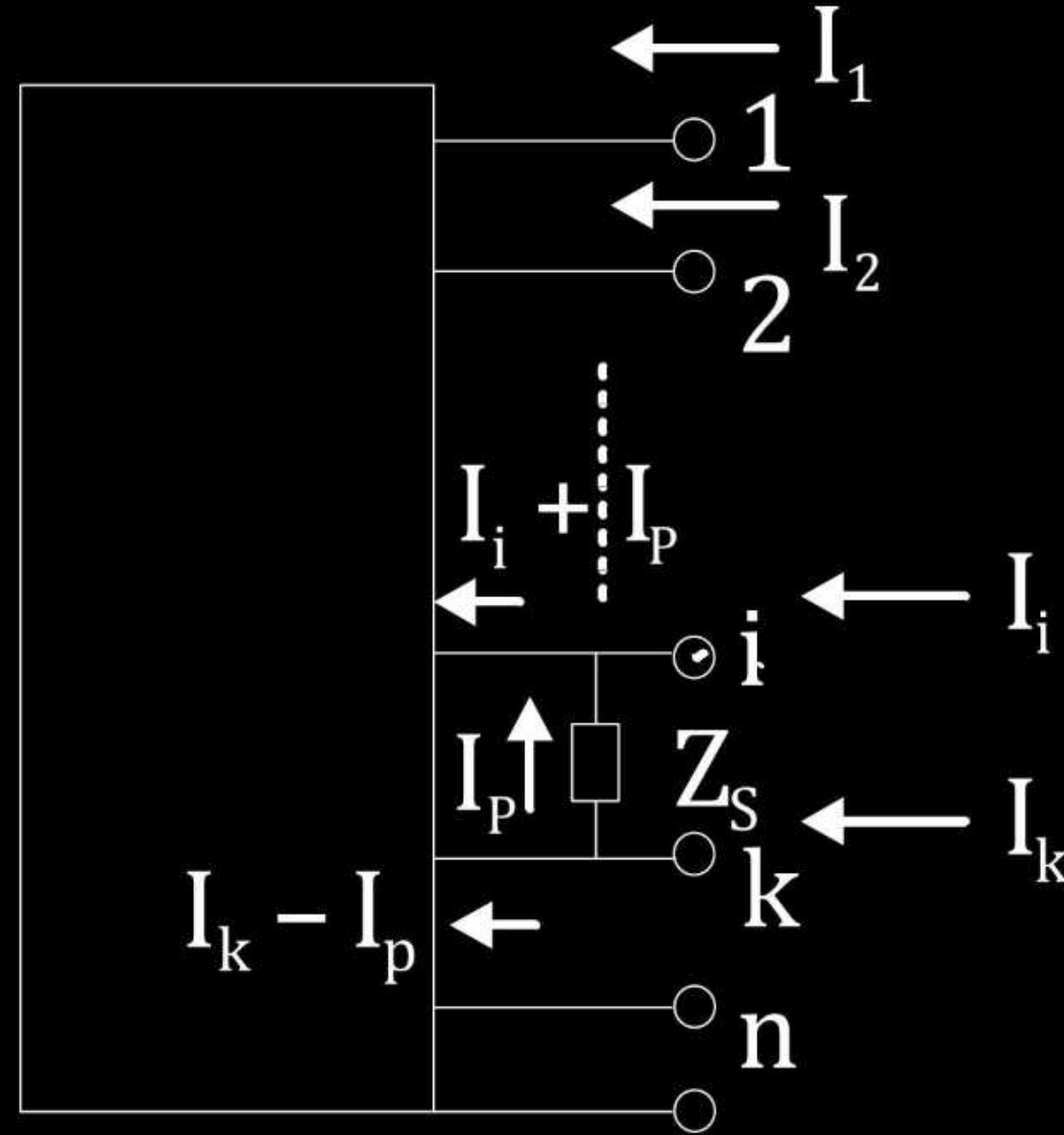
$$\begin{bmatrix}
 - & - & - & - \\
 - & j0.3408 & j2586 & - \\
 - & - & - & - \\
 - & - & - & -
 \end{bmatrix}
 - \frac{1}{j5408}
 \begin{bmatrix}
 - \\
 j3408 \\
 - \\
 -
 \end{bmatrix}
 \begin{bmatrix}
 - & - & - & - \\
 - & j3408 & j2586 & 0 \\
 - & - & - & - \\
 - & - & - & -
 \end{bmatrix}$$

$$Z_{22}(ncw) = j0.3408 - \frac{1}{j0.5408} (j0.3408 \times j0.3408) = j0.1260$$

$$Z_{23}(ncw) = j0.2586 - \frac{1}{j0.5408} [0.3408 \times 0.2586]$$
$$= \underline{0.0956}$$

Type 4: New element Z_s is added between bus i & k existing

$$V_i + I_p Z_s = V_k$$



$$\left. \begin{aligned} I_i &= I_i + I_p \\ I_k &= I_k - I_p \end{aligned} \right\}$$

Reference

We connect a element Z_s between i & k and we assume Z_s as current in Z_s is I_p

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \dots + Z_{1j}(I_j + I_p) \dots + Z_{1k}(I_k - I_p) + \dots + Z_{1n}I_n$$

$$\rightarrow V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \dots + Z_{1j}I_j + \dots - Z_{1k}I_k + \dots - Z_{1n}I_n + (Z_{1j} - Z_{1k})I_p \quad \text{--- ①}$$

$$\rightarrow V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \dots + Z_{2j}I_j + \dots - Z_{2k}I_k + \dots - Z_{2n}I_n + (Z_{2j} - Z_{2k})I_p \quad \text{--- ②}$$

$$\rightarrow V_j = Z_{j1}I_1 + Z_{j2}I_2 \quad \dots + Z_{jj}I_j + \dots - Z_{jk}I_k + \dots - Z_{jn}I_n + (Z_{jj} - Z_{jk})I_p \quad \text{--- ③}$$

$$\rightarrow V_k = Z_{k1}I_1 + Z_{k2}I_2 \quad \dots + Z_{kj}I_j \quad \dots + Z_{kk}I_k + \dots - Z_{kn}I_n + (Z_{kj} - Z_{kk})I_p \quad \text{--- ④}$$

$$\rightarrow V_n = Z_{n1}I_1 + Z_{n2}I_2 \quad \dots + Z_{nj}I_j + \dots + Z_{nk}I_k + \dots - Z_{nn}I_n + (Z_{nj} - Z_{nk})I_p \quad \text{--- ⑤}$$

$$\rightarrow V_j + I_p Z_s = V_k$$

$$Z_{j1}I_1 + Z_{j2}I_2 \dots + Z_{ji}I_i + \dots + Z_{jk}I_k \dots + Z_{jn}I_n + (Z_{ji} - Z_{jk})I_p \overset{+I_p Z_s}{-}$$

$$= Z_{k1}I_1 + Z_{k2}I_2 \dots + Z_{ki}I_i + \dots + Z_{kk}I_k \dots + Z_{kn}I_n + (Z_{ki} - Z_{kk})I_p$$

$$\rightarrow 0 = (Z_{j1} - Z_{k1})I_1 + (Z_{j2} - Z_{k2})I_2 + \dots + (Z_{ji} - Z_{ki})I_i + \dots + (Z_{jk} - Z_{kk})I_k$$

$$+ (Z_{jn} - Z_{kn})I_n + (Z_{ji} - Z_{jk} - Z_{ki} + Z_{kk} + Z_s)I_p \text{--- ⑥}$$

$$\begin{bmatrix}
 v_1 \\
 v_2 \\
 \vdots \\
 v_j \\
 \vdots \\
 v_k \\
 \vdots \\
 v_n \\
 0
 \end{bmatrix}
 \begin{bmatrix}
 z_{11} & z_{12} & \dots & z_{1j} & \dots & z_{1k} & \dots & z_{1n} & (z_{1j} - z_{1k}) \\
 z_{21} & z_{22} & \dots & z_{2j} & \dots & z_{2k} & \dots & z_{2n} & (z_{2j} - z_{2k}) \\
 \vdots & \vdots & & \vdots & & \vdots & & \vdots & \\
 z_{j1} & z_{j2} & \dots & z_{jj} & \dots & z_{jk} & \dots & z_{jn} & (z_{jj} - z_{jk}) \\
 \vdots & \vdots & & \vdots & & \vdots & & \vdots & \\
 z_{k1} & z_{k2} & \dots & z_{kj} & \dots & z_{kk} & \dots & z_{kn} & (z_{kj} - z_{kk}) \\
 \vdots & \vdots & & \vdots & & \vdots & & \vdots & \\
 z_{n1} & z_{n2} & \dots & z_{nj} & \dots & z_{nk} & \dots & z_{nn} & (z_{nj} - z_{nk}) \\
 \hline
 (z_{jj} - z_{k1}) & (z_{jj} - z_{k2}) & (z_{jj} - z_{kj}) & (z_{jk} - z_{kk}) & \dots & (z_{jj} - z_{jk} - z_{kj} + z_{kk} + z_s)
 \end{bmatrix}$$

$$\begin{bmatrix}
 I_1 \\
 I_2 \\
 \vdots \\
 I_j \\
 \vdots \\
 I_k \\
 \vdots \\
 I_n \\
 I_p
 \end{bmatrix}$$

Now eliminate $(n + 1)$ row & $(n + 1)$ column.

$$[Z_{Bus}]_{new} = [Z_{Bus}]_{old} - \frac{1}{\underbrace{Z_s + Z_{ii} + Z_{kk} - 2Z_{ki}}_{(Row\ i - Row\ k)}} \begin{matrix} \overset{\text{Column } i - \text{Column } k}{\begin{bmatrix} Z_{1i} & -Z_{1k} \\ Z_{2i} & -Z_{2k} \\ \vdots & \vdots \\ Z_{ni} & -Z_{nk} \end{bmatrix}} \end{matrix}$$

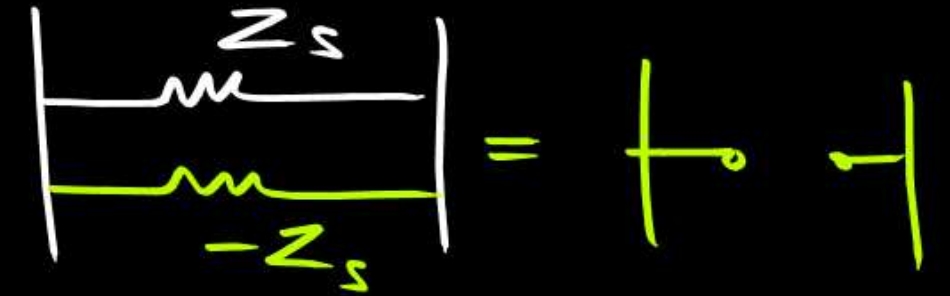
$$[(Z_{i1} - Z_{k1})(Z_{i2} - Z_{k2}) \cdots (Z_{in} - Z_{kn})]$$

$$Z_{ii} + Z_{kk} - 2Z_{ki} + Z_s$$

Removing a branch/changing impedance of any element

→ A single branch of impedance Z_s can be removed by addition of new element of $-Z_s$ between same node

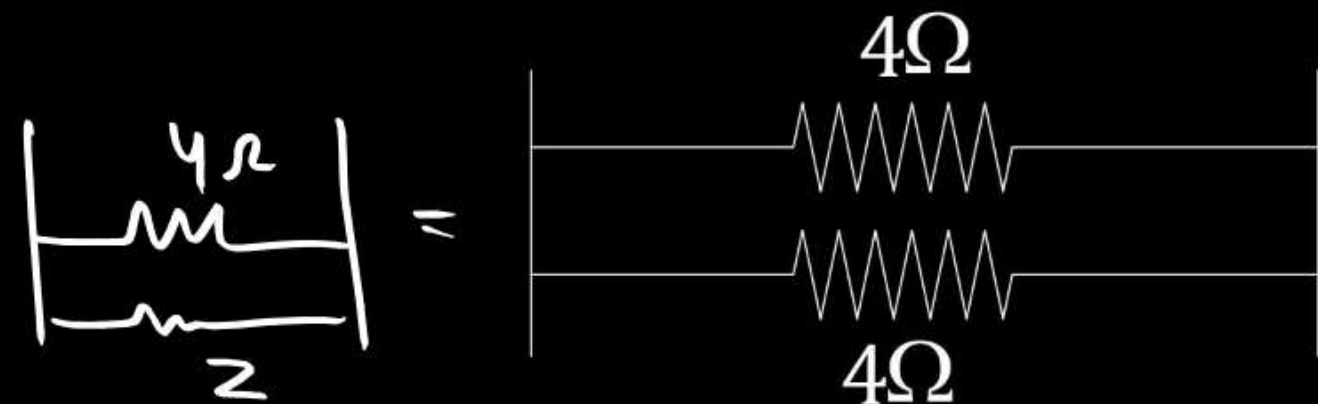
$$\frac{Z_s \times (-Z_s)}{Z_s - Z_s} = \infty \quad (\text{Open circuit})$$



If impedance of any element is to be change. Add a element (Impedance) in parallel of existing impedance such that their equation impedance is equal to the desired value.

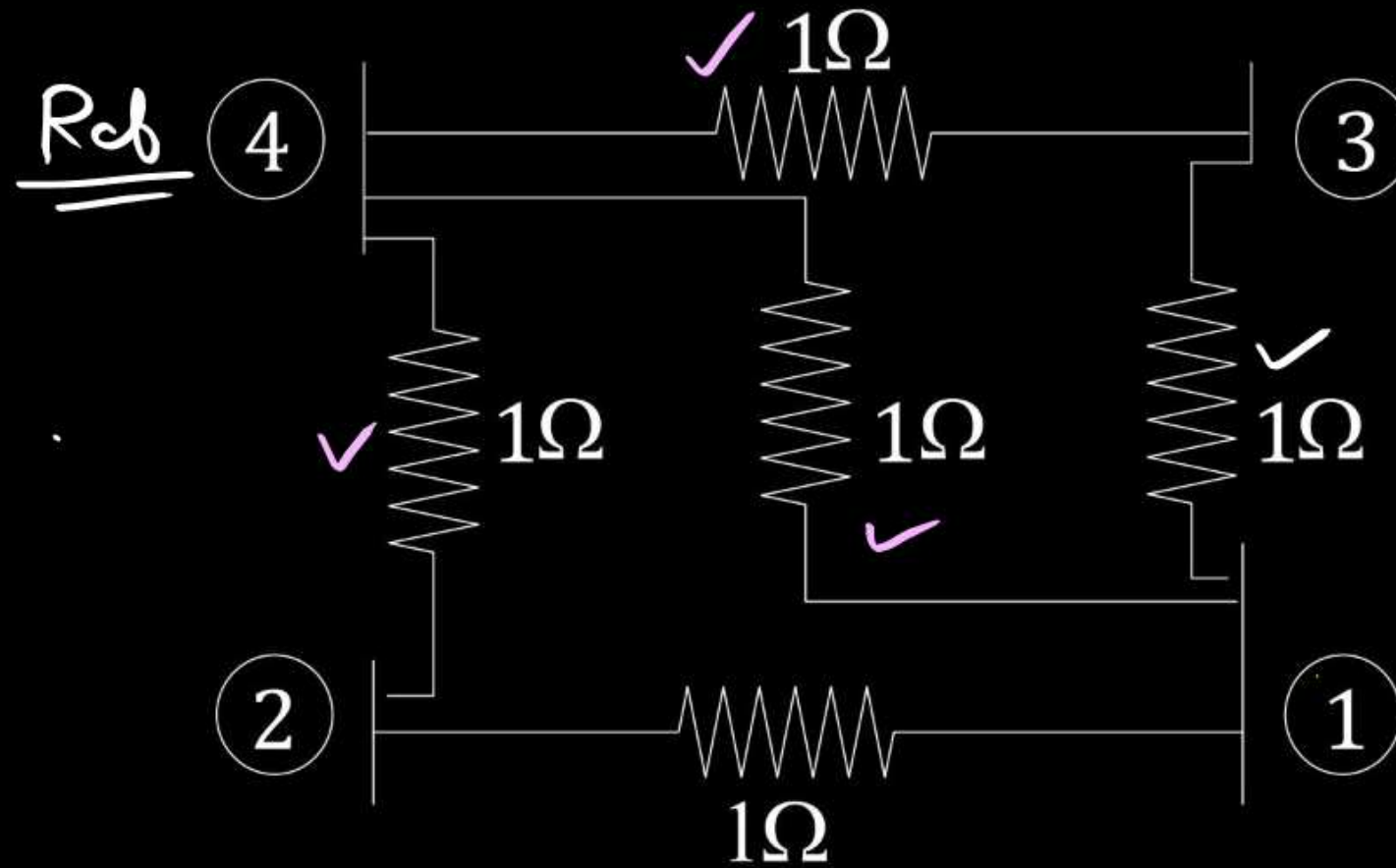
$$4 \rightarrow 2$$

$$\frac{4 \times 2}{4 + 2} = \underline{\text{Desire}}$$



Example

Form Z_{Bus} by treating bus 4 as a reference bus



Step 1

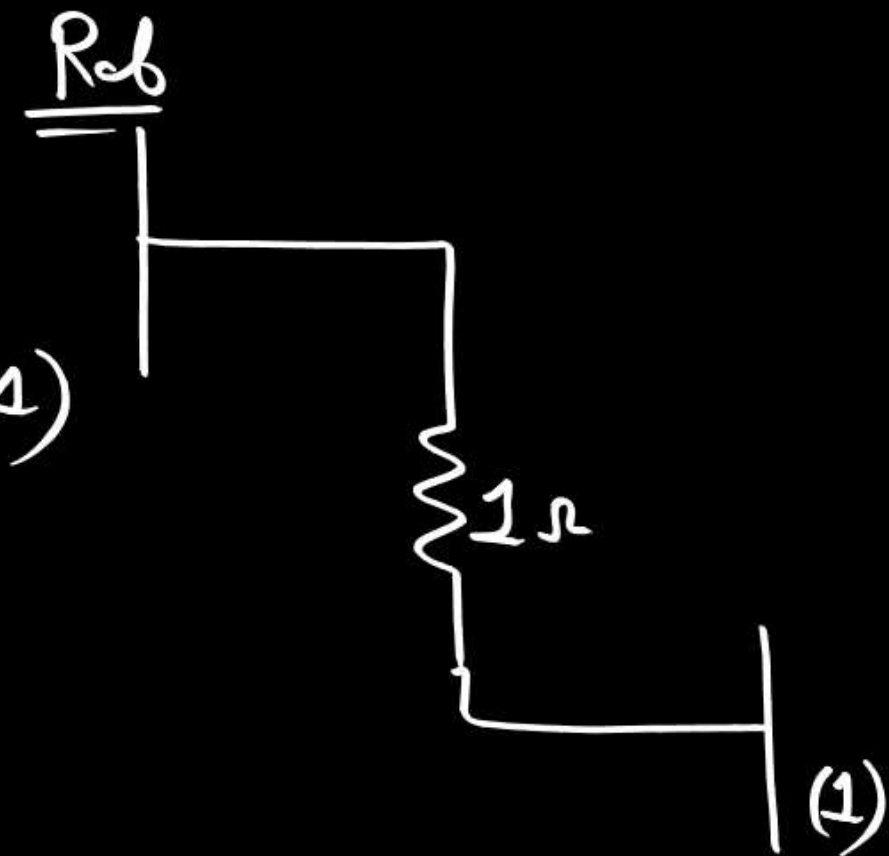
No Bus exist in power system so

$$Z_{Bus} = \text{NIL}$$

Step-2

Adding new element $Z_s = 1\Omega$ Between new Bus(1) and Reference Bus.

$$Z_{Bus} = [1]$$



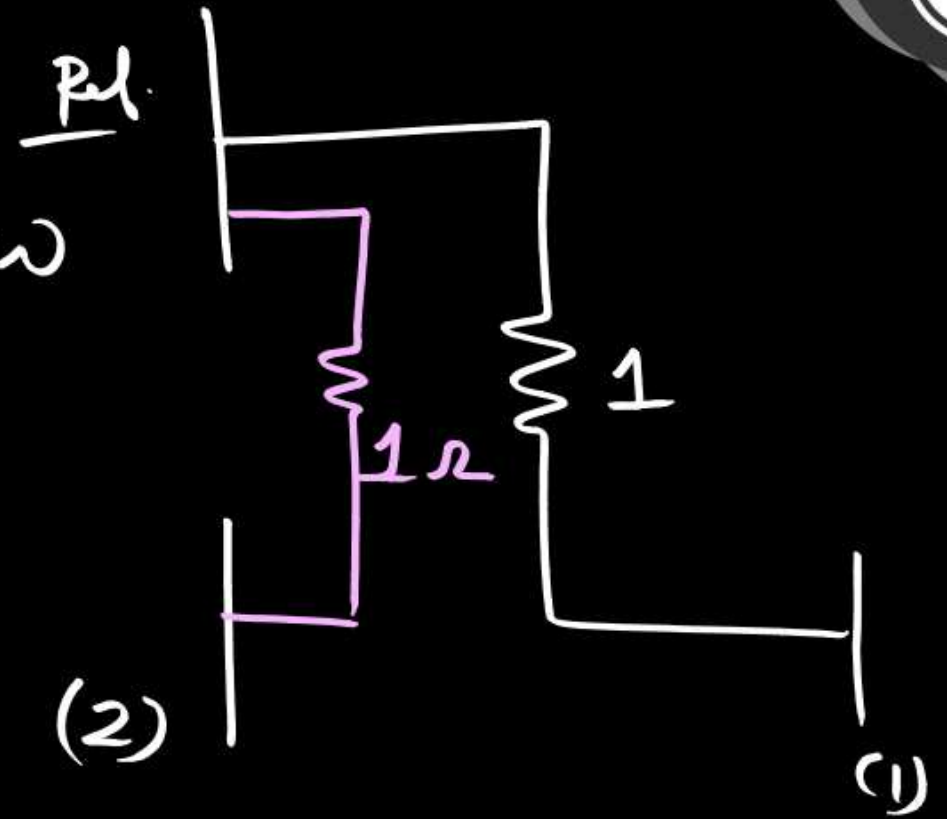
Step 3

Adding new element $Z_s = 1\Omega$ between new Bus (2) and Ref Bus

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step 4

Adding new element $Z_s = 1\Omega$ between new Bus 3 and reference Bus.

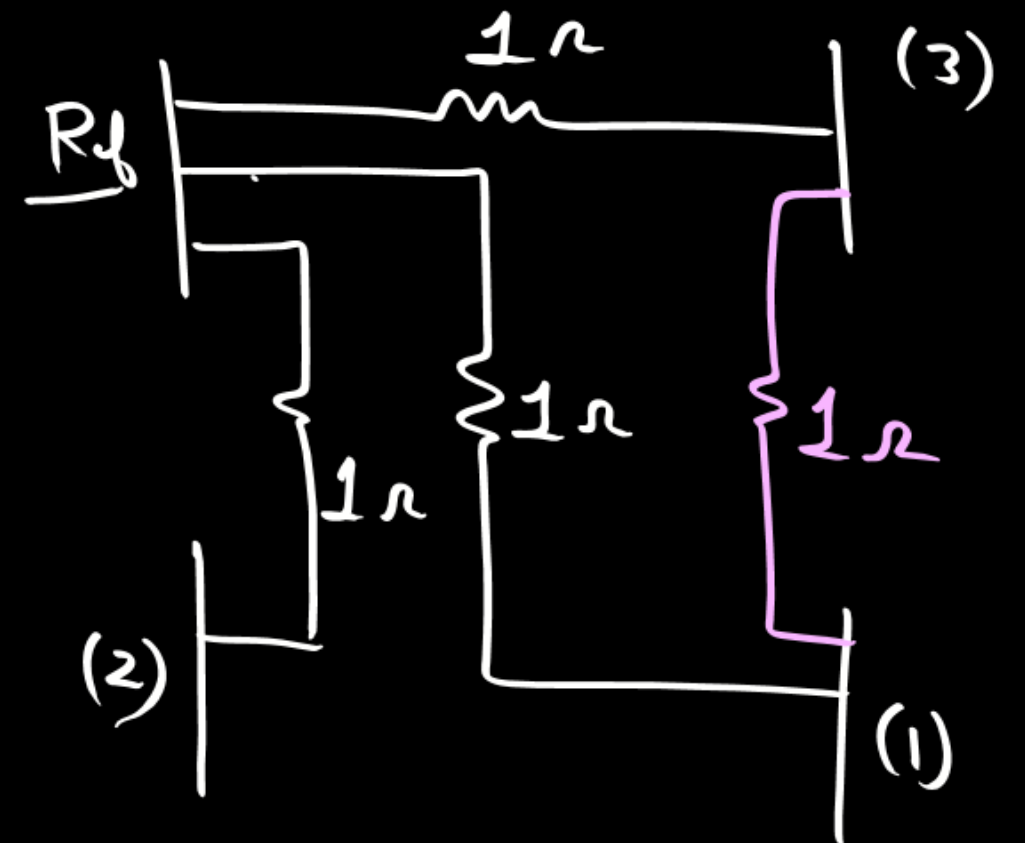
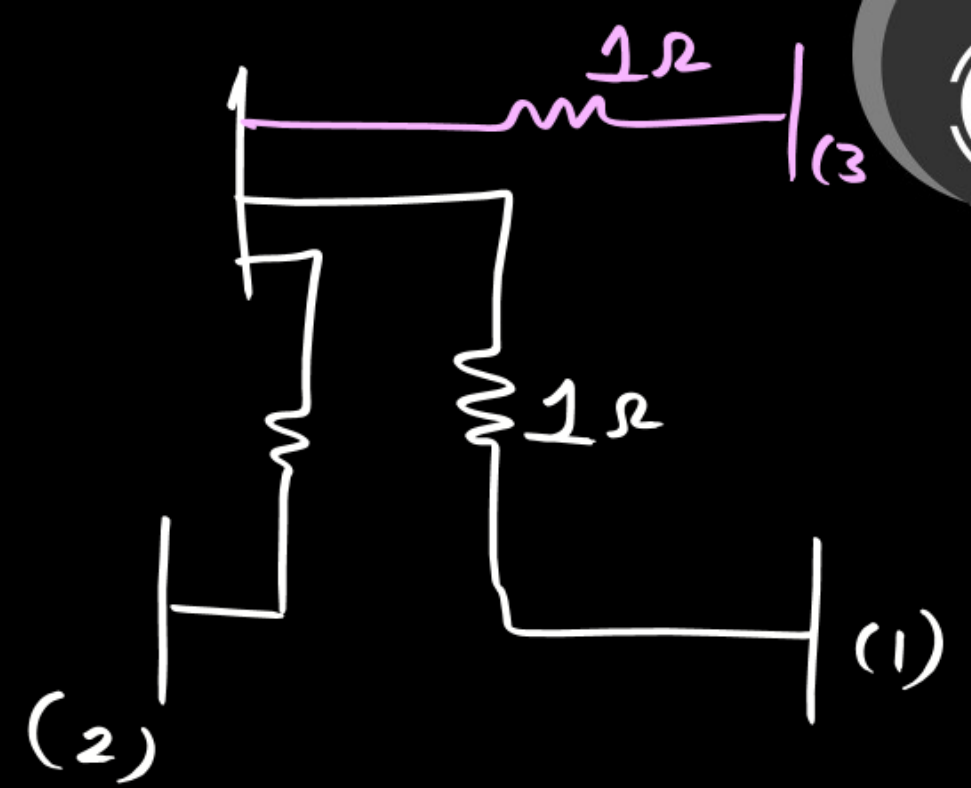


$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array} \right]$$

Step-4

An element $Z_s = 1$ is connected between existing Bus $i=1$ and $k=3$

$$\begin{aligned} & Z_{ij} + Z_{kk} - 2Z_{ik} + Z_s \\ &= Z_{11} + Z_{33} - 2Z_{13} + Z_s \end{aligned}$$



$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{1+1-2(0)+1} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

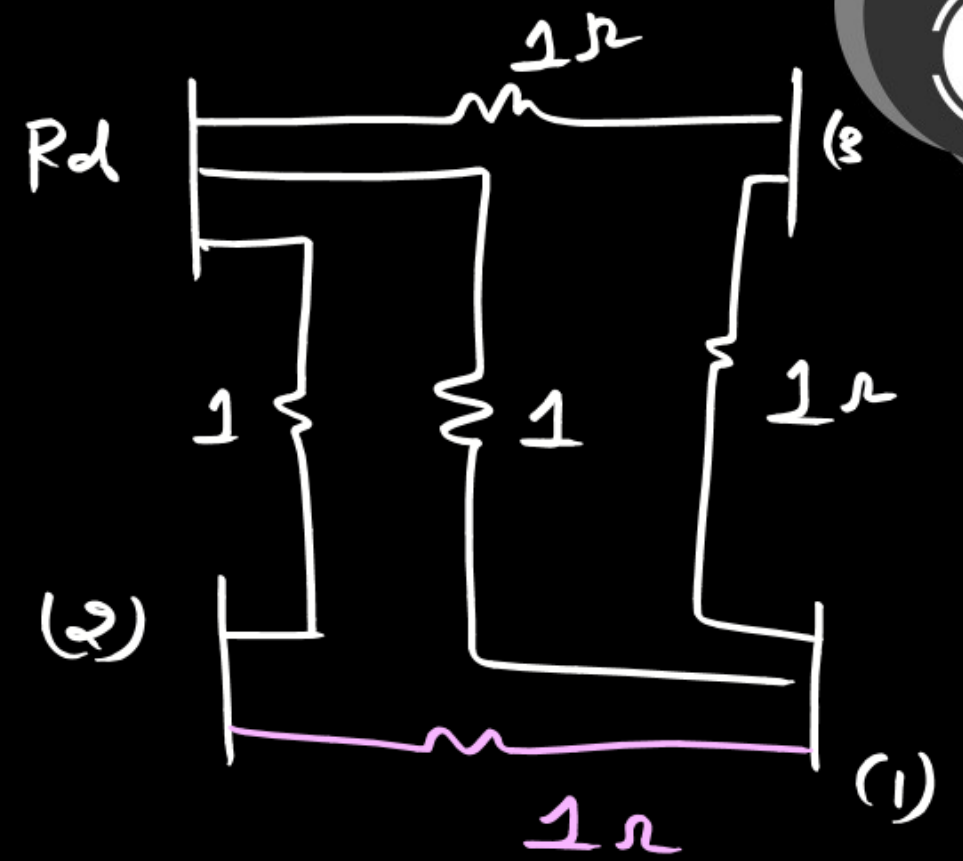
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix}$$

Step-5

An element $Z_s = 1$ is connected Between existing Bus $i=1$ & $k=2$

$$Z_{ii} + Z_{kk} - 2Z_{ik} + Z_s$$

$$Z_{11} + Z_{22} - 2Z_{12} + Z_s$$

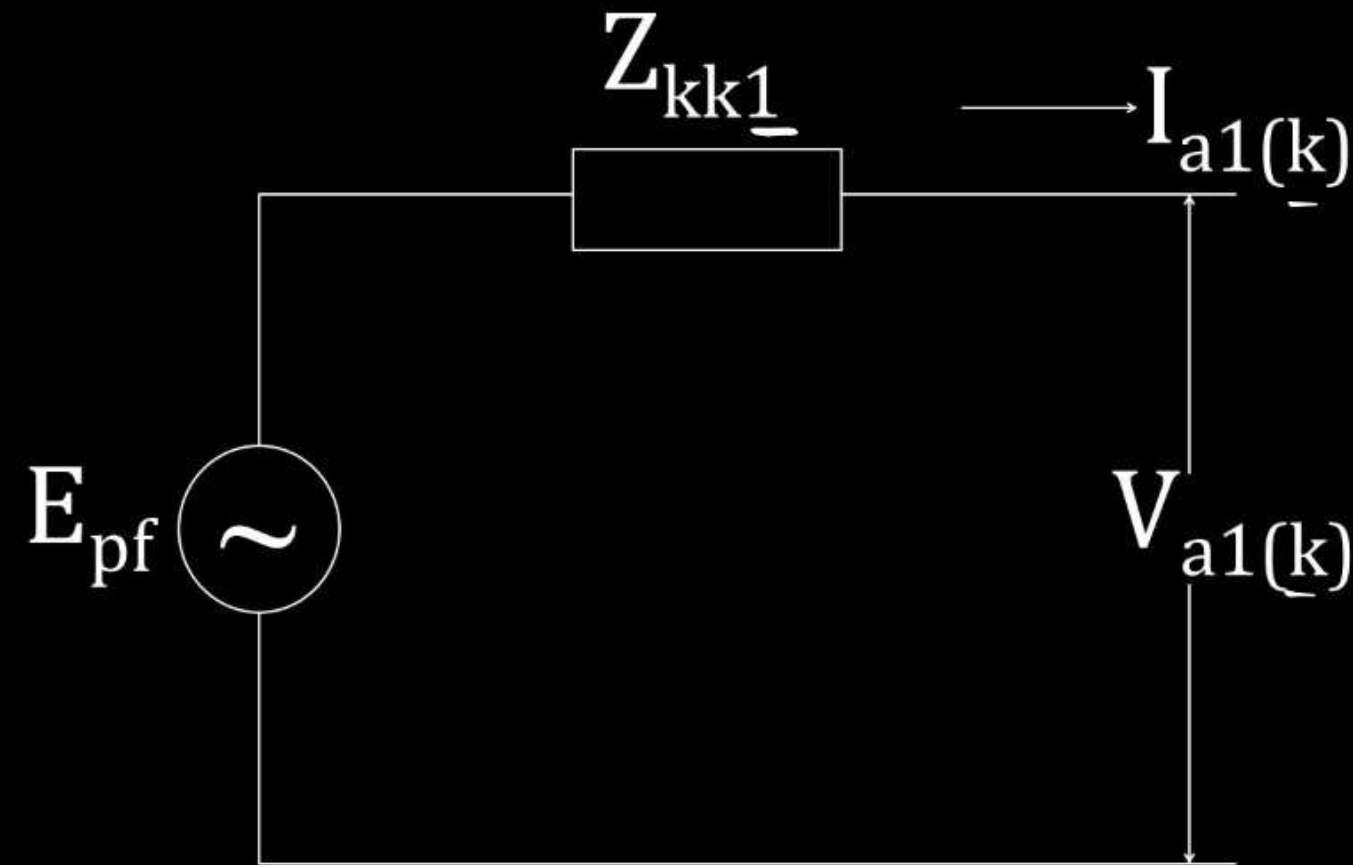


$$\begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ -\frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix} - \frac{1}{\frac{2}{3} + 1 - 2(0) + 1} \begin{bmatrix} \frac{2}{3} \\ -1 \\ \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -1 & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix} - \frac{3}{8} \begin{bmatrix} \frac{4}{9} & -\frac{2}{3} & \frac{2}{9} \\ -\frac{2}{3} & 1 & -\frac{1}{3} \\ \frac{2}{9} & -\frac{1}{3} & -\frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{5}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{5}{8} \end{bmatrix}$$

Fault Analysis Using Z Matrix

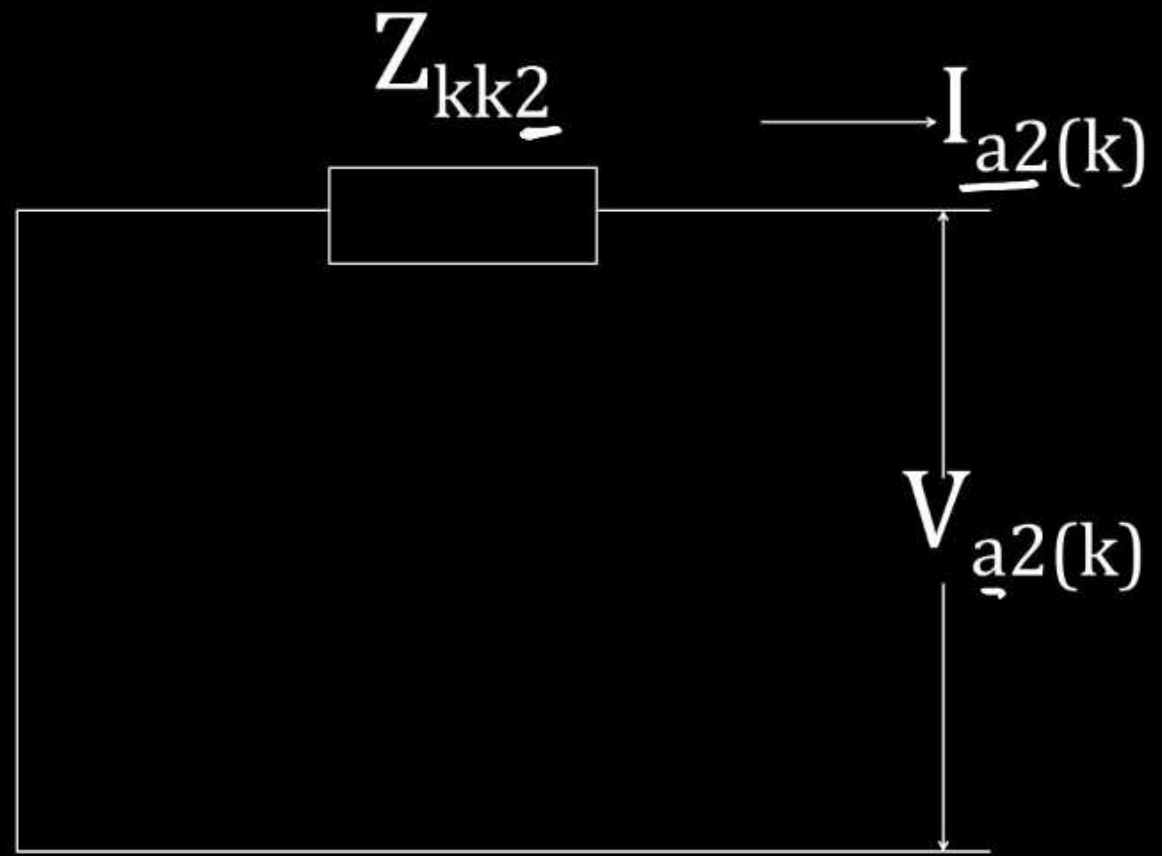
1. For positive, negative and zero sequence network Z bus matrix is formed.
2. The Thevenin equivalent circuit between fault point (k^{th} Bus) and reference can be drawn as shown in figure below.



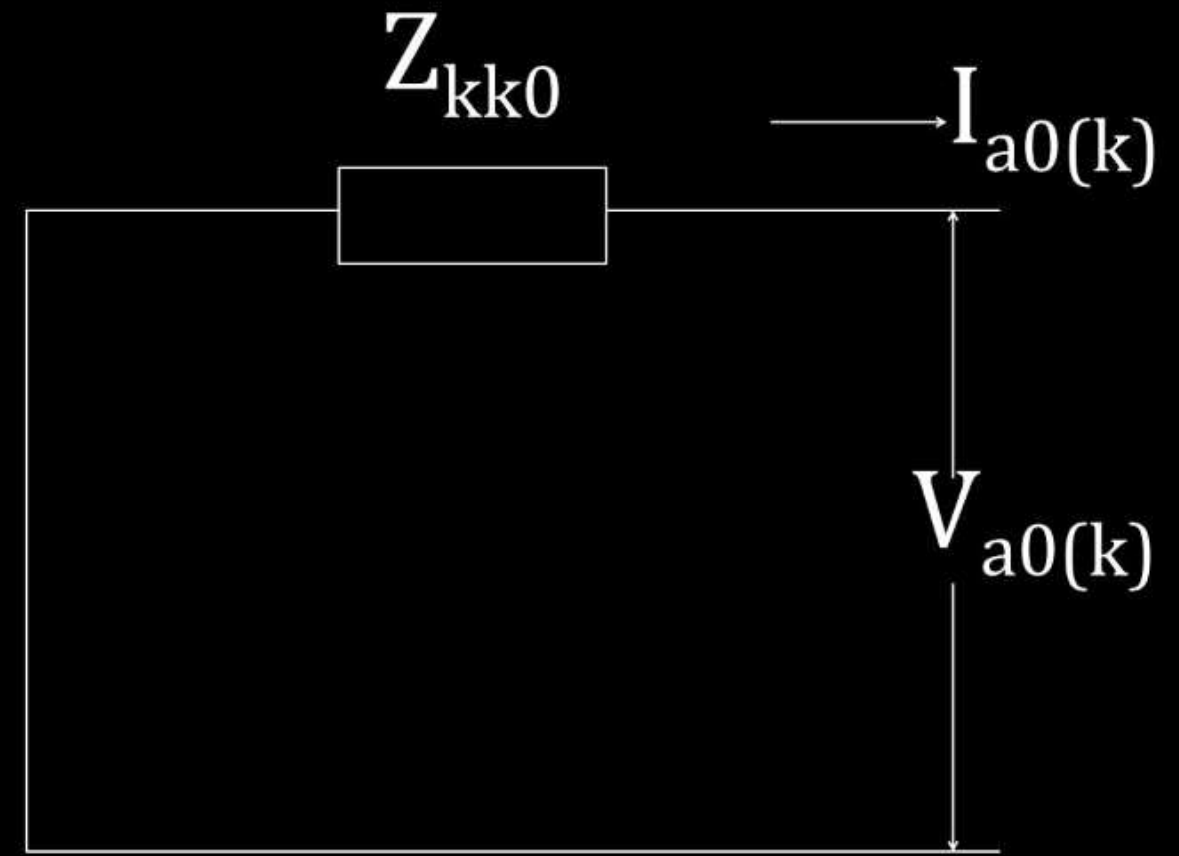
Positive Sequence Network

Z_{11}
 Z_{22}
 Z_{33}

Negative Sequence Network



Zero Sequence Network





Impedance matrix of positive square network from P.S.N

$$\begin{bmatrix} Z_{11}(1) & Z_{12}(1) & Z_{13}(1) \\ Z_{21}(1) & Z_{22}(1) & Z_{23}(1) \\ Z_{31}(1) & Z_{32}(1) & Z_{33}(1) \end{bmatrix}$$

Impedance matrix of Negative square network from N.S.N

$$\begin{bmatrix} Z_{11}(2) & Z_{12}(2) & Z_{13}(2) \\ Z_{21}(2) & Z_{22}(2) & Z_{23}(2) \\ Z_{31}(2) & Z_{32}(2) & Z_{33}(2) \end{bmatrix}$$



Impedance matrix of zero square network from ZSN

$$\begin{bmatrix} Z_{11}(0) & Z_{12}(0) & Z_{13}(0) \\ Z_{21}(0) & Z_{22}(0) & Z_{23}(0) \\ Z_{31}(0) & Z_{32}(0) & Z_{33}(0) \end{bmatrix}$$

3. These 3 Thevenin equivalent circuit are connected together as per type of fault.



L-L-L Fault(3 phase fault)

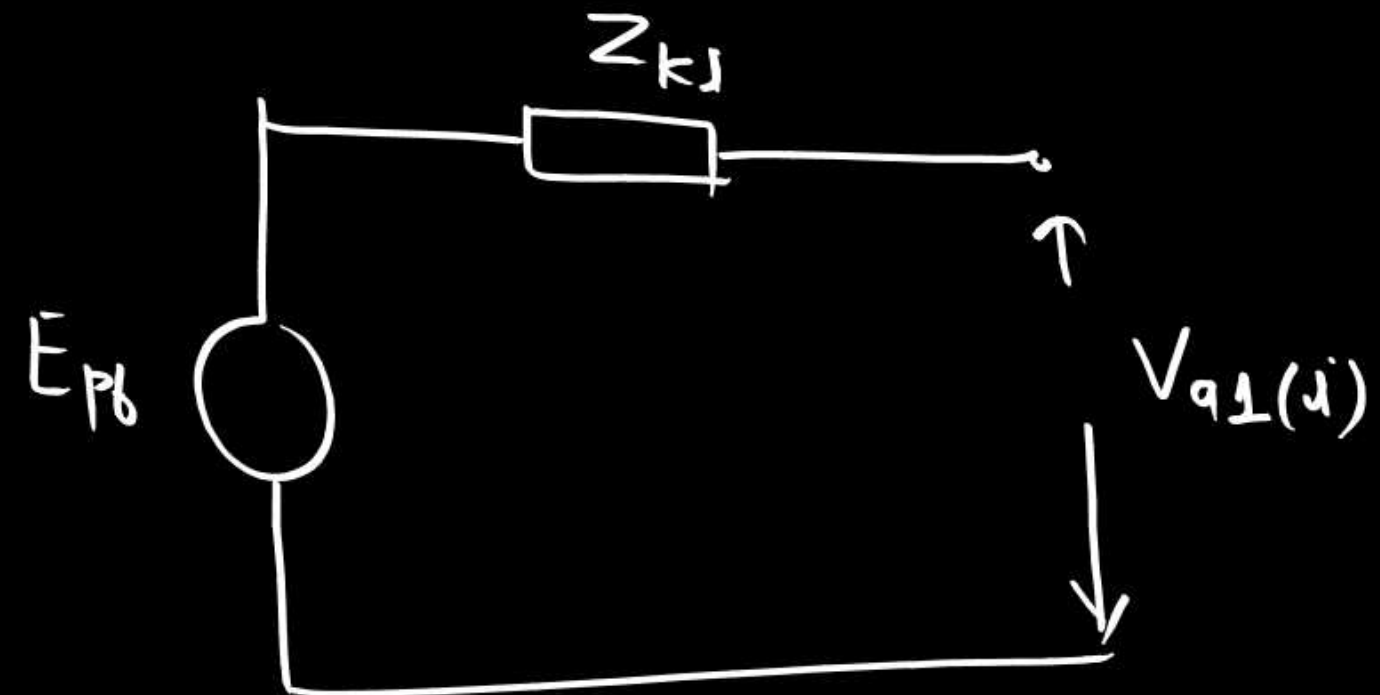
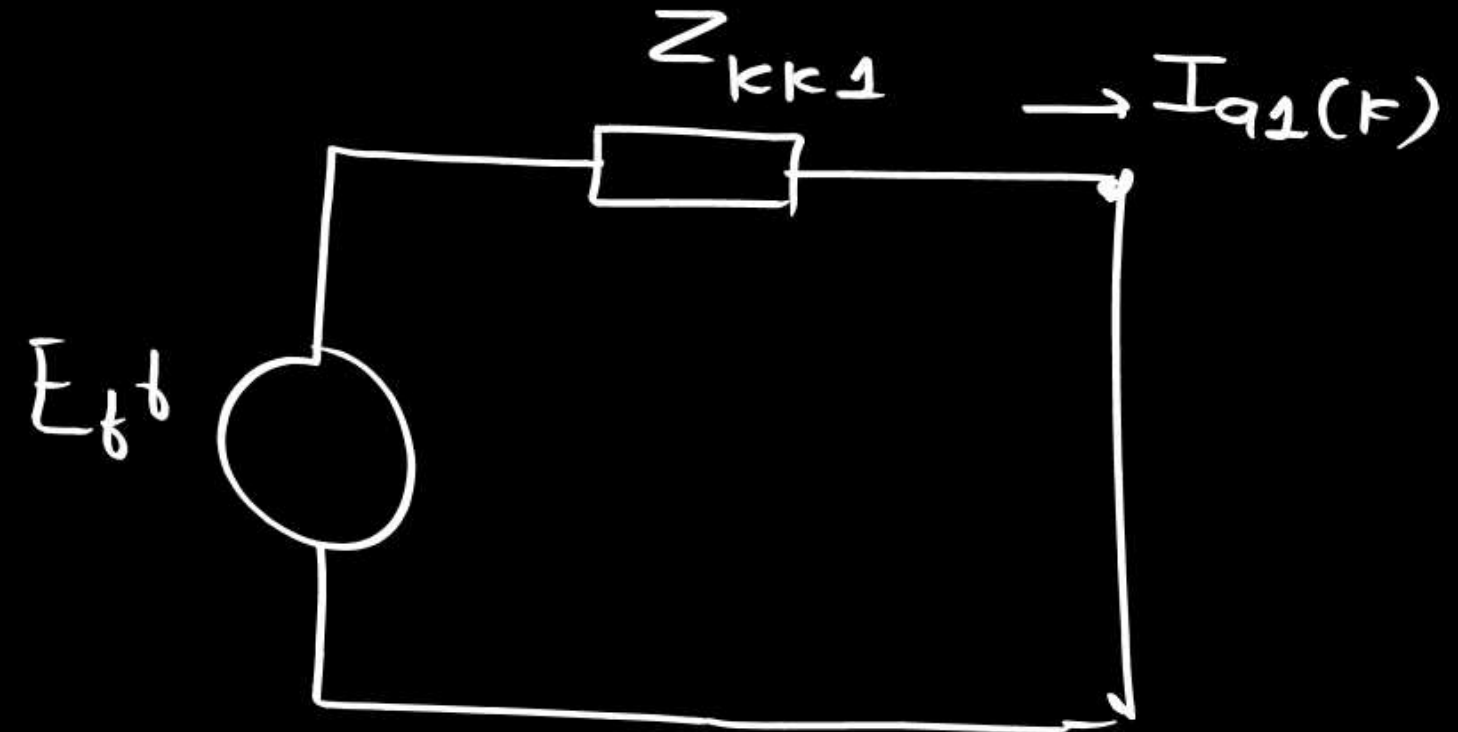
$$\text{➤ } I_{a1(k)} = \frac{E_{pf}}{Z_{kk1} + Z_f}$$

➤ Voltage at i^{th} bus

$$V_{a1(i)} = E_{pf} - Z_{ik1} I_{a1(k)}$$

$$V_{a2(i)} = -Z_{ik2} I_{a2(k)}$$

$$V_{a0(i)} = -Z_{ik0} I_{a0(k)}$$



Voltage at i^{th} bus

$$V_{a0(i)} = -Z_{ik0} I_{a0(k)}$$

$$V_{a1(i)} = E_{pf} - Z_{ik1} I_{a1(k)}$$

$$V_{a2(i)} = -Z_{ik2} I_{a2(k)}$$

L-G Fault



$$I_{a1(k)} = \frac{E_{pf}}{Z_{kk0} + Z_{kk1} + Z_{kk2} + Z_f}$$

$$I_f = 3(I_{a1k})$$

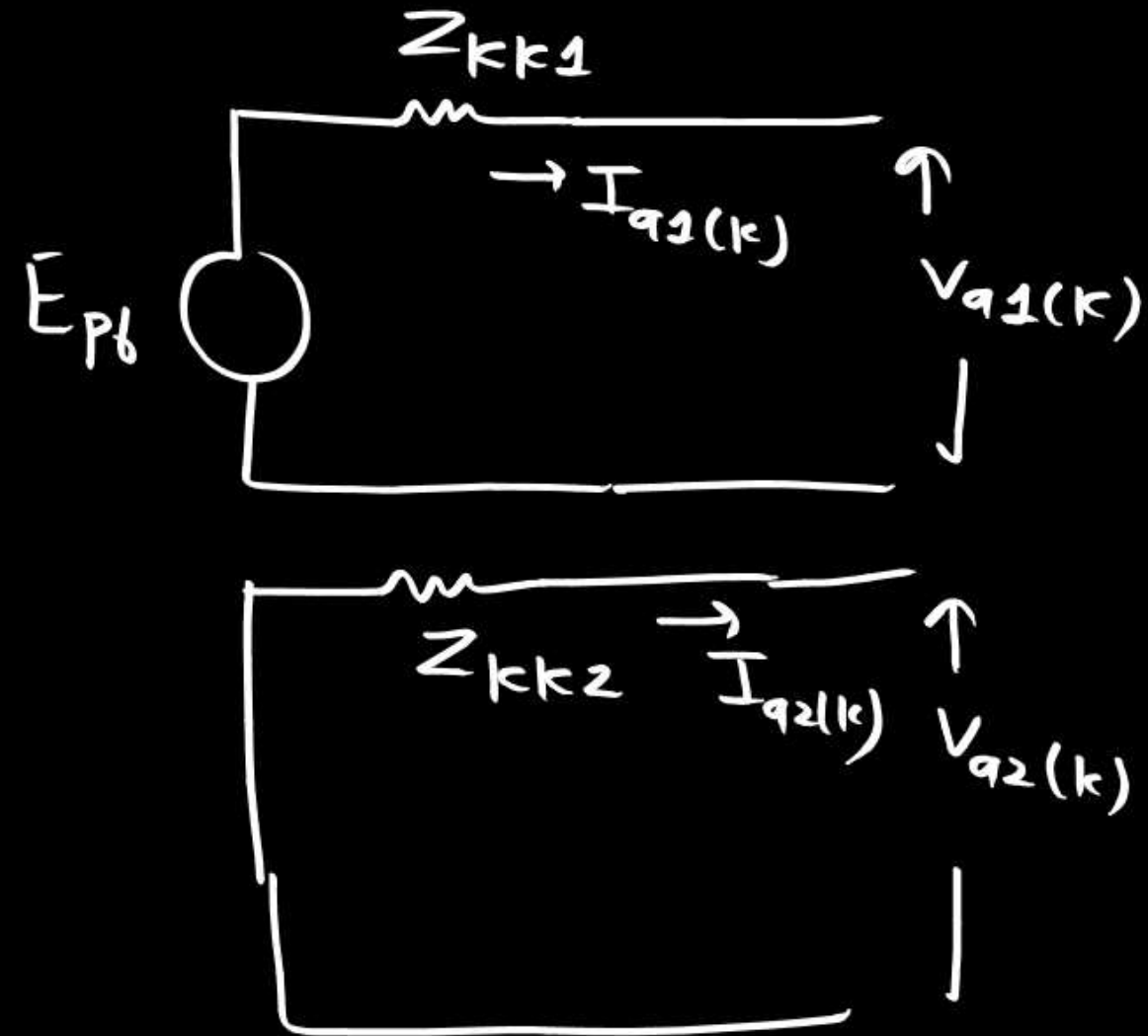
Voltage at Bus k

$$V_{a1(k)} = \bar{E}_{pf} - I_{a1(k)} Z_{kk1}$$

$$V_{a2(k)} = -I_{a2(k)} Z_{kk2}$$

$$V_{a0(k)} = -I_{a0(k)} Z_{kk0}$$

$$I_{a1(k)} = I_{a2(k)} = I_{a0(k)}$$

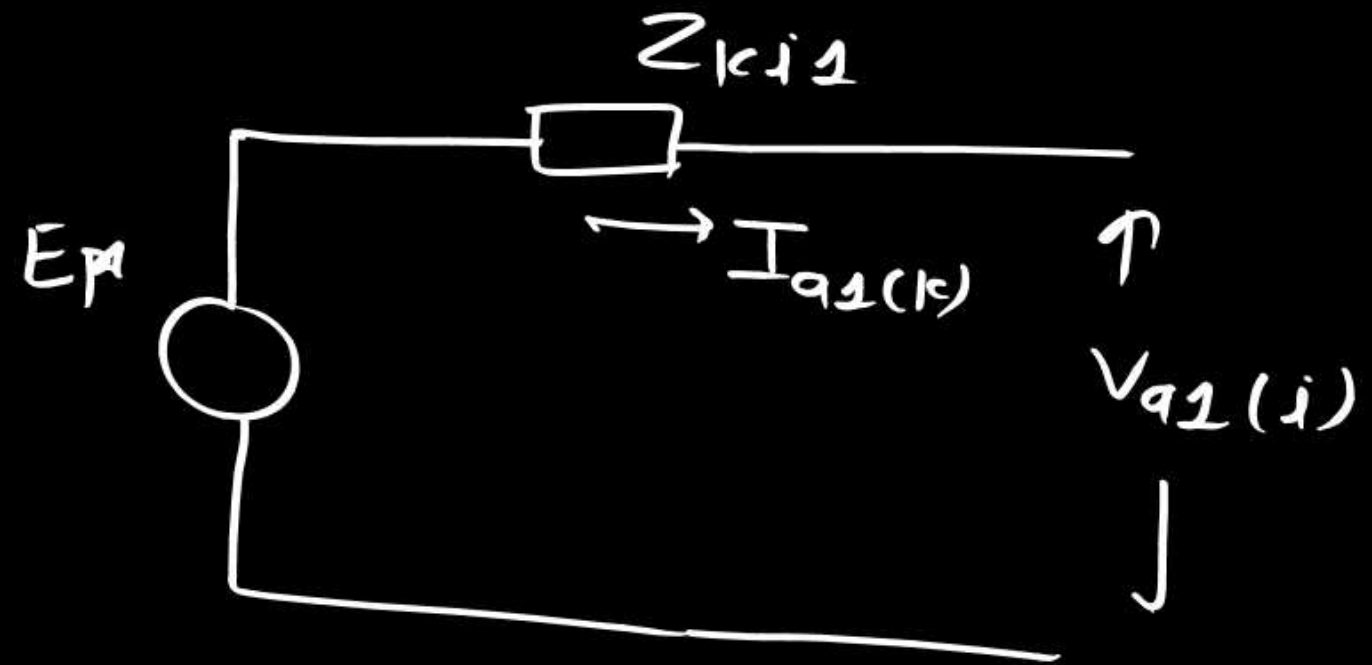


Voltage at i^{th} bus

$$V_{a0(i)} = -\widetilde{Z}_{ik0} I_{a0(k)}$$

$$V_{a1(i)} = E_{pf} - \widetilde{Z}_{ik1} I_{a1(k)}$$

$$V_{a2(i)} = -\widetilde{Z}_{ik2} I_{a2(k)}$$



Line to Line Fault



$$I_{a1(k)} = \frac{E_{pf}}{Z_{kk1} + Z_{kk2} + Z_f}$$

$$I_{a2(k)} = -I_{a1(k)}$$

$$I_f = -j\sqrt{3}I_{a1(k)}$$

Voltage at k Bus

$$V_{a1(k)} = E_{pf} - Z_{kk1}I_{a1(k)}$$

$$V_{a2(k)} = -Z_{kk2}I_{a2(k)}$$

$$V_{a0(k)} = -Z_{kk0}I_{a0(k)}$$

Voltage at i^{th} bus

$$V_{a0(i)} = -Z_{\underline{ik}0} I_{a0(k)}$$

$$V_{a1(i)} = E_{pf} - Z_{ik1} I_{a1(k)}$$

$$V_{a2(i)} = -Z_{ik2} I_{a2(k)}$$

Double Line to Ground Fault



$$I_{a1(k)} = \frac{E_{pf}}{Z_{kk1} + \frac{Z_{KK2} + (Z_{KK0} + 3Z_f)}{Z_{kk2} + Z_{KK0} + 3Z_f}}$$

$$I_f = 3I_{a0(k)}$$

Voltage at i Bus

$$V_{a1(i)} = E_{pf} - Z_{ik1}I_{a1(k)}$$

$$V_{a2(i)} = -Z_{ik2}I_{a2(k)}$$

$$V_{a0(i)} = -Z_{ik0}I_{a0(k)}$$

Voltage at k Bus

$$V_{a1(k)} = E_{pf} - Z_{kk1}I_{a1(k)}$$

$$V_{a2(k)} = -Z_{kk2}I_{a2(k)}$$

$$V_{a0(k)} = -Z_{kk0}I_{a0(k)}$$

[NAT]



#Q. The bus impedance matrix of a 3-bus system (in pu) is

$$Z_{bus} = \begin{bmatrix} j0.059 & j0.061 & j0.038 \\ j0.061 & j0.093 & j0.066 \\ j0.038 & j0.066 & j0.110 \end{bmatrix}$$

A symmetrical fault (through a fault impedance of $j0.007$ pu) occurs at bus 2. k =

Neglecting pre-fault loading conditions, the voltage at bus 1, during the fault is _____ pu (round off to three decimal places).

[1 Mark]

$$j = 1$$

$$V_{a1}(j) = E_{pf} - I_{a1}(k) Z_{jk}$$

$$I_{a1(2)} \frac{1}{Z_{kk} + Z_t} = \frac{1}{Z_{22} + Z_t} = \frac{1}{j0.093 + j0.007} = -j10$$

$$\begin{aligned} V_{a1(1)} &= E_{P_t} - I_{a1(2)} Z_{12} \\ &= 1 - (-j10)(j0.061) \\ &= 1 - 0.61 \\ &= \underline{0.39} \end{aligned}$$



2 Minutes Summary

→ Modification of Imp Matrix

→ fault Analysis using Impedance matrix

Thank
you